The problem of treasury portfolio optimisation

Participants of the project:
Alexey Kulemin, Vasily Sorokoumov, Vasily Konuzin, Victor Kulyshhev
Moscow 2002
The Savings bank of the Russian federation is the largest financial institution in Russia that accumulates about 70% of the retail holdings and 30% of the corporate holdings. It is a true universal bank providing its services through a four-level chain of branches spread throughout Russian Federation.

The Treasury of the Savings bank is the largest market participant on the internal markets. It makes about 20% of MISEX daily volumes on FOREX section and it completely dominates over Government debt sector.
Optimization tasks:

- Eurobond portfolio optimization
- Stock portfolio optimization
- Currency portfolio optimization
- Precious metals portfolio optimization
The currency portfolio of the Treasury of the Savings Bank of the Russian Federation has certain specifics conditioned by the banks’ unique position in Russia:

- Largest client bank
- Largest government agent on the market

This results in:

- The largest number of transactions driven thru Sberbank
- Necessity to hold positions in variety of currencies
- Necessity to hold large reserves
- Necessity to optimize currency portfolio
The primary goal of the research was to determine the efficient portfolio frontier and construct the optimal currency portfolio, which would provide an investor with

a). **Maximum expected return** at a given level of risk
b). **Minimum risk** at a given level of the expected return
Currency portfolio

Having the efficient frontier calculated trader is able to view the current structure of the optimal portfolio and the point corresponding to his portfolio, thus he is able to make decisions regarding his policy and actions necessary.

- On the one hand he may consider a conservative policy and move to the left upper corner of the efficient frontier, which significantly reduces risks, however diminishes profit.
- On the other hand move to the right upper corner of the efficient portfolio frontier will bring risks into portfolio mainly associated with high currency volatilities.
Assumptions:

Spot rates only

Optimal portfolio does not include any long-term instruments, such as deposits, swaps and forwards. Exclusion of interest rate instruments allowed narrowing the task to one factor problem, where the main determinant of the risk is the FX rate.

Restriction on open deals

Another assumption is that optimal portfolio has no open deals, i.e. the deals started previously with value date in the future. For example a swap deal with first leg yesterday and second day in one week.

Absolute values of positions

To measure the value of the observed open currency position short and long positions were taken as absolute values and summarized.
Market data for the research

Closing bids since December 1999 for currencies composing the portfolio since December 1999 were considered (more than 500 observations for each currency). The following time series of the FX rate were taken:

<table>
<thead>
<tr>
<th>Hard currencies</th>
<th>Soft currencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Britain Pound</td>
<td>Australian Dollar</td>
</tr>
<tr>
<td>Euro</td>
<td>Canadian Dollar</td>
</tr>
<tr>
<td>Swiss Frank</td>
<td>Byelorussia ruble</td>
</tr>
<tr>
<td>Deutsche Mark</td>
<td>Czech crone</td>
</tr>
<tr>
<td>French Franc</td>
<td>Kazakh tenge</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>Poland zloty</td>
</tr>
<tr>
<td></td>
<td>Russian ruble</td>
</tr>
<tr>
<td></td>
<td>Danish crone</td>
</tr>
<tr>
<td></td>
<td>Norwegian crone</td>
</tr>
<tr>
<td></td>
<td>Singapore dollar</td>
</tr>
<tr>
<td></td>
<td>Swedish crone</td>
</tr>
</tbody>
</table>

At the second step real currency positions data was taken for finding current currency portfolio on *the expected return – risk plain*. 
Expected return calculation

FX rates were used to calculate volatility of currencies, corresponding VaR, and the expected rate of return. The expected return was found as:

\[ Y = \frac{P_n - P_{n-1}}{P_{n-1}} \]

where

- \( Y \) – expected return,
- \( P_n \) – FX rate at a day \( n \)
- \( P_{n-1} \) – FX rate at a day \( n-1 \)

The expected return was calculated for three different horizons: 1 day, 1 week and 1 month.
Risk calculation
One of the important issues of the research was comparison of different techniques of risk calculation and examining time intervals and data series on which those techniques work best. As possible risk factors FX rates were considered. The risk for each currency was calculated using four methods presented in SAS Risk Dimensions:

- Risk Metrics® (Delta Normal)
- Historical simulation
- Monte-Carlo using covariance matrix
- Monte Carlo using fitted models
Distribution of profit and loss of the currency portfolio calculated using historical simulation (left) and Monte Carlo technique (right).

Distribution of profit and loss for currency portfolio calculated with the help of Monte Carlo technique using fitted models.
The solution of Markowitz problem with the restricted open position has provided efficient portfolio frontier, allowing viewing the structure of the portfolio at each point on the *expected return – risk plain*.

![Graph showing efficient portfolio frontier and current portfolio position]

- Efficient portfolio frontier
- Current portfolio position

**Legend:**
- **Yield:** %
- **VaR:** %
- **RM - RiskMetrics®**
- **MR - Monte Carlo**
- **HS - Historical simulation**
- **MC_FM - Monte Carlo using fitted models**
Research showed that the optimal currency portfolio providing the minimum level of risk must consist of the following currencies: Deutsche marks, euro, Kazakh tange. Adding Canadian dollars, rubles, Japanese yens and Swedish crones would increase the portfolio profitability.
Conclusions re: Currency portfolio optimization

- Similar results of VaR calculation
- Implementation of fundamental optimization models to currency market
- VaR calculation for each currency and for overall currency portfolio

<table>
<thead>
<tr>
<th>Backtesting results for money market</th>
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</thead>
<tbody>
<tr>
<td>Risk Metrics</td>
</tr>
<tr>
<td>Confidence intervals</td>
</tr>
<tr>
<td>Exceed values</td>
</tr>
</tbody>
</table>
Conclusions re: Currency portfolio optimization

• Sensitivity analysis

- Statistical estimates
- Internationally accepted risk reporting
- Advanced analytical solutions development
Eurobonds’ portfolio

For the past few years speculative operations on FOREX market were extremely attractive for market participants because of relatively low risks they provided as well as higher margins compared to other segments of the Russian financial market.

Currently the expected return of currency market is about 5,5% per annum (LIBOR), at the same time, average internal rate of return on eurobonds is 10%. Without a doubt such difference makes investment in securities more attractive:

- traded over-the-counter, thus a larger number of market participants have a possibility to take part in trades.
- issuer of these securities is government, which makes them the least risky Russian debt.
- nominated in foreign currencies, in particular American dollars, which have lower inflation rates and larger circulation on international markets, therefore providing higher stability and liquidity for securities.
Eurobonds’ portfolio

Often the situation on the security market is strongly dependent upon the policy of Sberbank of Russia this is the reason for the Sberbank viewing of the market both from profit and stability preserving sides.

Currently one of the main streams in the development of banking technology in Sberbank of Russia is a search for suitable analytical solution allowing risk management at both enterprise and desk levels.

Targets of the Eurobond portfolio optimization were not constrained by calculating risks and returns, but also included analysis of the market in terms of profitability, liquidity, attractiveness of making investments, as well as finding current position of Sberbank eurobond portfolio on the risk-expected return plain and moving it towards the efficient portfolio frontier.
## Eurobonds’ portfolio

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<th>Coupon frequency</th>
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<th>Currency</th>
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<tr>
<td>03.06.98</td>
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<td>Eurobond 03</td>
<td>0.1175</td>
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<td>1000000</td>
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<td>2</td>
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</tr>
</tbody>
</table>
Calculation of an interest rate curve

Scenario as applied to the selected base case curve

Russian eurobonds yield curve

Credit spread for Russian federation

USD zero coupon curve

Base Case Curve [Government Zero curve for USD = (usd_govzero), using EB_Current]

Scenario Curve
Pricing function

In order to calculate present value of each instrument in the portfolio, cash flows of each instrument were discounted. Pricing function in that case looked as follows:

\[
MtMvalue = \sum_{i=1}^{N} \frac{c_{f_i} r_i}{(1 + \frac{r_i}{m})^{i*m}} + \frac{FaceValue}{(1 + \frac{r_N}{m})^{N*m}}
\]

*MtMvalue* – market to market value of a bond
*cf* – coupon payments
*FaceValue* – bond par value
\(r_i\) – interest rate, corresponding to coupon payment date (taken from the constructed yield curve)
\(N\) – number of the last payment
\(m\) – number of coupon payment per year.
Eurobonds’ portfolio

VaR models

Similarly to the currency portfolio optimization task the risks were calculated using VaR approach. Again,

- Risk Metrics® (Delta Normal)
- Historical simulation
- Monte-Carlo using covariance matrix
- Monte Carlo using fitted models

were used for risk measurement
Eurobonds’ portfolio

Profit/Loss distribution using Historical Simulation

Profit/Loss distribution using Monte Carlo Simulation

Distribution by type of bonds

Profit/Loss distribution using Monte Carlo with fitted models
Eurobonds’ portfolio

The basis towards making investment according to the modern portfolio formation theory is H. Markowitz work.

Taking into account mostly investment-oriented nature of the Sberbank portfolio of bonds, Markowitz optimization model could be used as a ground for conducting a further research.

Consequently the optimization task was narrowed down to finding the possible portfolios frontier and positioning of the current Sberbank portfolio on the risk-expected return plain.
Eurobonds’ portfolio

Efficient portfolio frontier

Optimal portfolio structure
Conclusions re: Eurobonds’ optimization task

• Results of VaR calculation

VaR by types of bonds

VaR by instruments
Eurobonds’ portfolio

Conclusions re: Eurobonds’ optimization task

BackTesting Results for Eurobonds
Eurobonds’ portfolio

Conclusions re: Eurobonds’ optimization task

• Results of VaR calculation

Spreads used for eurobond yield curve calculation influenced the overall portfolio sensitivity the most.
Conclusions re: Eurobonds’ optimization task

• Optimization task solution
• Portfolio modeling
• International reporting standards
• Statistical estimates
• Advanced analytical solutions development
• Optimization task solution for the model stock portfolio
For the last time the stock market causes more and more careful attention of investors, which is described by prevalent tendency of the securities market development. For the past months stock-leaders were constantly demonstrating stable growth.

Stock market offers the largest margins, taking the lead among other segment of securities market. In some cases expected return of stocks runs up to 3-4% daily, which is about 1000% annually, while at the same time, eurobond or currency expected return amounts 10% and 5.5% respectively.

In the atmosphere of universal interest towards stock market, banks should develop its risk evaluation and management conceptions and consequently aim at finding the optimal ratio between risks and the expected return of the portfolio.
As the largest universal bank, Sberbank of Russia could not be aside from the problems of expansion of its presence on the stock market and, at the same time, optimization of related operations and positions.

The research equipped specialized software for risk evaluation and management at enterprise level – SAS/Risk Dimensions, developed by SAS Institute. Actually this particular research was a continuation and development of projects aimed at evaluation of risks of currency, eurobonds and precious metals portfolios.

Therefore, solution of this task would also allow making substantial judgments regarding contribution of risk and profit each position of an overall bank portfolio makes.
## Stock portfolio

### Input data:

<table>
<thead>
<tr>
<th>Name</th>
<th>ID</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lukoil</td>
<td>LUKOIL</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>UKOS</td>
<td>UKOS_AO_2</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Surgutneftegaz</td>
<td>SURGNFGZ</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Sibneft</td>
<td>SIBN</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>RAO UES</td>
<td>RAOEES</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Norilski Nikel</td>
<td>NORNIK</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Tatneft</td>
<td>TATNFT</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Rostelekom</td>
<td>ROSTEL_1AO</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Mosenergo</td>
<td>MOSENERGO</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Sberbank of Russia</td>
<td>SBERBANK</td>
<td>30 000 000 rub.</td>
</tr>
<tr>
<td>Gazprom</td>
<td>GAZP</td>
<td>30 000 000 rub.</td>
</tr>
</tbody>
</table>
Leading positions on the Russian stock market still belong to oil-and-gas processing industry (6 out of 10), at the same time they bring the most profits and have the largest market capitalization.

Overall capitalization of RTS is $59 164 945 146,00, while capitalization of enterprises of oil-and-gas processing industry amounts $48 468 192 614,00 (82% of an overall market capitalization).

Therefore, conclusion about strong dependency of Russian equity market upon the situation on the international commodity market can be made.

It is important, that even slight decrease in prices for resources may reflect in significant fall in processing and extracting industry enterprises stock prices, which in turn, would decrease portfolio value aloud.
In order to optimize the stock portfolio, the original Markowitz problem was applied to the market data and results of risk calculation.
Conclusions re: optimization task for stocks

The research demonstrated that foreign optimization models can be applied to Russian equity market, provided they are adopted to more dynamic and instability of market conditions. Markowitz optimization model was applied to the portfolio consisting of top-ten leaders of the Russian trading system and Gazprom stocks, traded on MFB. Results of the solution provided weight of instruments necessary for optimal portfolio achievement.

Unfortunately Markowitz model assumes forming of the investment portfolio, i.e. the portfolio remains constant during the observed in the research period: sum of the portfolio weight equals one. Moreover, the model also assumes ban on the short position. As a result of the above said, Markowitz model cannot be used as it is for forming speculative portfolio. Additional research should be conducted in order to find necessary solution.
Conclusions re: optimization task for stocks

VaR estimates, computed using different models, were obtained through the research. Risk Dimensions provided VaR estimates via different analytical sections, preset by user, with the help of cross-classifications.
Estimate of each risk factor influence on the portfolio risk was obtained when calculating risk via Monte Carlo methods. Sensitivity of the portfolio to the particular risk factor was calculated on the base of covariance matrix, automatically generated by Risk Dimensions during VaR calculation process.
Conclusions re: optimization task for stocks

Assuming that the risk and expected return values are known, the optimal portfolio may be constructed. Optimum criteria can be complicated in compliance with the bank’s strategic goals. For example it is possible to input additional restrictions on portfolio diversification degree, like share of particular securities should not be larger than “…”, or less than ”…” and so on. During the optimal portfolio construction both expert and fundamental evaluations could be used.

Significant positive correlation between stock quotes provides a possibility for construction of an aggregate index for Russian stocks. Its construction requires additional research driven on extended range of input data. Furthermore, positive cross-correlation coefficients witnesses the impossibility of straight hedging of the stock portfolio. However selecting the optimal portfolios on the efficient portfolio frontier can reduce risks of the portfolio.
Precious metals portfolio

The optimization solution for currency portfolio was used as a ground for development similar solution for precious metals portfolio. The major task was to examine the current distribution of weights among the metals composing the portfolio. The adjustment of the currency solution for precious metals portfolio took one week only.

The structure of the observed portfolio is shown in the table bellow:

<table>
<thead>
<tr>
<th>MetalName</th>
<th>BaseDate</th>
<th>MtM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>20.02.02</td>
<td>250 008,00</td>
</tr>
<tr>
<td>Au</td>
<td>20.02.02</td>
<td>250 002,01</td>
</tr>
<tr>
<td>Pd</td>
<td>20.02.02</td>
<td>249 750,00</td>
</tr>
<tr>
<td>Pt</td>
<td>20.02.02</td>
<td>250 380,00</td>
</tr>
<tr>
<td>Total</td>
<td>20.02.02</td>
<td>1 000 140,01</td>
</tr>
</tbody>
</table>
Precious metals portfolio

 Metals Portfolio Optimal Curve

 Metals Portfolio Structure

yield
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03

VeR
0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

-0.03 -0.02 -0.01 0 0.01 0.02 0.03 0.04 0.05

Au  Ag  Pt  Pd
The Eurobonds Portfolio Optimization


Vasiliy E. Sorokoumov, Alexey A. Kulemin, Vasilii V. Konuzin, Victor Y. Kulyshev
Savings Bank of Russian Federation, Moscow, Russia, 2002.

1. Introduction

For the past few years speculative operations on FOREX market were extremely attractive for market participants because of relatively low risks they provided as well as higher margins compared to other segments of the Russian financial market. Such situation can be explained by nearly complete absence of the capital market after the crisis of August 1998. However, in the last two years security market has been demonstrating a stable tendency of growth, which refers both to equity and fixed income markets.

Currently the expected return of currency market is about 5.5% per annum (LIBOR), at the same time, average internal rate of return on eurobonds is 10%. Without a doubt such difference makes investment in securities more attractive. The government bonds where taken as an example not accidentally for they are the most liquid and stable securities circulating on the market. This can be explain by the fact that government debt, nominated in the foreign currencies is generally traded over-the-counter, thus a larger number of market participants have a possibility to take part in trades. The second thing is that the issuer of these securities is government, which makes them the least risky Russian debt. Finally eurobonds are nominated in foreign currencies, in particular American dollars, which have lower inflation rates and larger circulation on international markets, therefore providing higher stability and liquidity for securities.

The Savings bank of Russian Federation is the largest market participant on the Russian debt market. Often the situation on the security market is strongly dependent upon the policy of Sberbank of Russia this is the reason for the Sberbank viewing of the market both from profit and stability preserving sides.

Though government debt market is considered to be the least risky within the country borders it is necessary to measure its risks, in order to form reserve capital, determine limits and of open position values more accurately. It is also important for a bank to disclose information about risks it faces, though it reflects in growing trust among its clients, as well as growing reference from its auditors. Often right asset and risk management model chosen by the bank may reflect in appropriation of a higher rating.

Disclosure of information regarding risks the financial institution faces and methods it uses to manage them will become more put more in weight within two-three years, due to the new requirements being prepared by Basel committee. Accordingly reserves the bank has to hold against its counter parties depend upon the credit rating of that counter party and its own rating. In such a way, provision of financial operation with the highly rated institution would require less back up. It is expected that banks themselves could develop models for credit scoring of counter parties. However those models must satisfy Basel committee requirements. This new regulation leaves financial institutions on their own – it is up to them to decide where to go – AAA or slide down. Important thing is that the credit rating of a sole bank could be higher than of a country it is registered in. from the western investment and credit institutions the most
riskless and profitable way of doing business is to provide services for higher rated institutions for it takes less money to be held in reserves. Conclusion that can be drawn is that running strategically right asset and risk management policy could improve the rating and increase turnover with western banks and other financial institutions.

2. Goals of the research

Currently one of the main streams in development of banking technology in Sberbank of Russia is a search for suitable analytical solution allowing risk management at both enterprise and desk levels. Optimization task of Russian debt nominated in US dollars was a part of the larger research aimed to find optimal ratio between risk and the expected return of the Sberbank’s treasury portfolio and determine the efficient strategy for resource redistribution between various segments of financial market.

Targets of the Eurobond portfolio optimization were not constrained by calculating risks and returns, but also included analysis of the market in terms of profitability, liquidity, attractiveness of making investments, as well as finding current position of Sberbank eurobond portfolio on the risk-expected return plain and moving it towards the efficient portfolio frontier.

As it was mentioned, in order solve optimization problem it is necessary to find an optimal ratio between the risk investor is ready to accept and the expected return he desires to get.

At the first stage risks of the portfolio were measured using different models: RiskMetrics®, historical simulation, Monte Carlo simulation, Monte Carlo simulation using fitted models. Results of risk calculation were then tested on adequacy (back-testing). Comparison of back-testing results could give an answer to what model predicts risks best, satisfies Basel committee requirements, and best suites the observed time series.

In order to solve the described risk management and optimization tasks, SAS/Risk Dimensions was chosen as one of the risk management leading solutions both on enterprise and desk levels. The project presumed active participation of both parties involved: Sberbank business and system analysts as well as specialists of the Russian office of SAS Institute.

3. Input data

Characteristics of securities chosen for the research are listed in the table below:

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<tr>
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<td>0.03</td>
<td>2</td>
<td>1000000</td>
<td>1</td>
<td>USD</td>
</tr>
</tbody>
</table>

4. Calculation of an interest rate curve

Interest rates are major risk factors for bonds, i.e. fluctuations of interest rates have the utmost impact on bonds’ value. Since observed securities were nominated in US dollars, US dollar deposit and forward rates were taken as risk factors for a time period less than half year and zero coupon yield curve was considered as a risk factor for a time period over half year2.

1 Eurobond 10 has a floating coupon rate. Table shows its average rate.
2 Reuters market data was used in the research
Combination of interest rate and zero coupon curves provided aggregated yield curve, which
describes the behavior of the market more accurately, because deposit and forward interest rates
quoted most actively in the half year period, thus represent market best on this interval, while
zero coupon curve illustrate it best on time intervals over half year.

However, the above aggregate curve could not be used for risk calculation since specifics
of Russian market (in particular country risk, market volume, profiles of its participants) reflect
in higher yield curve for Russian debt instruments nominated in US dollars compared to US
debt. The value of the exceed of Russian US dollar debt yield curve over American debt yield
curve (benchmark) is calculated and published by various information providers, in particular
case it was obtained from Reuters.

In order to construct the desired yield curve for Russian eurobonds, two benchmarks
were taken (10-year and 30-year), which linearly deformed yield curve (shifted above American
debt yield curve). As a result yields for Russian eurobonds were 4-5% higher than American
debt: the yearly yield on Russian eurobonds was about 6.5% versus 2% for American.
Conclusion drawn is that payment for risk of working with the Russian debt nominated in US
dollars is 4-5% per annum. It important to stress that benchmark values differ at the different
reference points, moreover benchmark is not a constant value and depending upon the situation
on the market changes.

![Figure 1.](image.png)

Figure 1. a). Linear deformation of the yield curve at 10 and 30-year reference point

b). Yield curve for American and Russian debt in US dollars

Since banking reporting is carried in rubles and observed eurobonds were nominated in
US dollars, additional risk factor – ruble-dollar exchange rate was added. In perspective this
would also allow recalculation of the research results both in dollars and rubles.

Value at Risk (VaR) estimate was used for portfolio and instrument risks illustration. As
it was mentioned, VaR was calculated using four different models: RiskMetrics®, historical
simulation, Monte Carlo simulation, Monte Carlo simulation using fitted models. Each model
has its own pro and contras and it is up to risk manager to decide which method best describes
current market situation and instruments in the portfolio.

5. **Pricing function**

In order to calculate present value of each instrument in the portfolio, cash flows of each
instrument was discounted. Discount rules were described in pricing function, which took into
account properties of instruments (date and value of coupon payments, principal repayments,
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e., discount rates for different time periods taken from the linearly deformed yield curve. Pricing function in that case looks as follows:

\[
MtMvalue = \sum_{i=1}^{N} \frac{cf_i}{(1 + \frac{r_i}{m})^{i*m}} + \frac{FaceValue}{(1 + \frac{r_N}{m})^{N*m}}
\]

where:
- \(MtMvalue\) – markt to market value of a bond
- \(cf\) – coupon payments
- \(FaceValue\) – bond par value
- \(r_i\) – interest rate, corresponding to coupon payment date (taken from the constructed yield curve)
- \(N\) – number of the last payment
- \(m\) – number of coupon payment per year.

6. VaR models

6.1 RiskMetrics®

As a matter of fact, RiskMetrics® is Delta-normal method enhanced by JP Morgan company. Essence of the model lies in risk calculation on the base of risk factor sensitivity. Sensitivity was found as differential of the pricing function.

RiskMetrics® model is relatively easy in calculation, does not require any powerful capacities for computation. However, the model assumes risk factors to be normally distributed. As is well known risk factors far often are not distributed normally which causes barriers for implementation of the particular method to some time series.

6.1 Historical simulation

The major assumption of historical simulation model is a complete representation of future scenarios of risk factor behavior by events occurred during the observed time period. This methodology proposes accumulation of risk factor changes over time interval, for example daily changes over past five years. It is assumed that the scenario sample of the past is a reliable representation of risk factors behavior on the forecasted time horizon and instruments consisting the portfolio are being repeatedly reprised for each scenario.

One of the major advantages of the model is that it does not restrict risk factor changes to any particular distribution. Hence, the described model provides reliable risk estimates independently upon the distribution lows.

At the same time model has several disadvantages. In order to implement the model the data on all observed risk factors should be collected, moreover it has be representative – i.e. deep enough to represent the past behavior of risk factors. Furthermore, scenarios used for VaR calculation, will be limited to events, which took place on the observed historical data sample. For example big distribution pips corresponding to the crisis of August 1998 are reflected in higher VaR estimate, and, consequently, all statistical estimates calculated on its basis will be affected also.

6.2 Monte Carlo simulation

Monte Carlo simulation is a methodology Monte Carlo simulation is a methodology to simulate scenarios for risk factor changes from any desired statistical distribution. The portfolio is re-valued at each simulated scenario, and a distribution of portfolio values is obtained. Ordering the changes in portfolio value from worst to best, the 99% VaR, for example, is computed as the loss such that 1% of the profits or losses are below it, and 99% are above it.

The key in meaningfully implementing Monte Carlo simulation is making reliable judgments about which statistical distribution is appropriate for which risk factors and estimating the parameters of the selected distributions. In practice, a wide array of distributions can be used
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for different risk factors. Some of the commonly used ones are the normal, the lognormal, GARCH, and so on. An important issue is the specification of a modeling structure that meaningfully takes into account the interrelationships between different risk factors.

It is instructive to compare Monte Carlo simulation with one characteristic of historical simulation. Monte Carlo simulation is capable of using the information in a historical sample about changes in risk factors to generate as many scenarios as requested. These scenarios are consistent with the statistical distribution that is judged to best describe the historical changes, but they are not limited to the exact scenarios that are observed in the historical sample, which is the case with historical simulation.

The main advantage of Monte Carlo simulation is that, if used appropriately in the right circumstances, it is capable of producing results that are the most accurate among all the VaR methodologies. Monte Carlo simulation is not constrained by the normality assumption, which applies to the delta normal method; nor by the limited scenario problem that was outlined above, which applies to historical simulation.

The most important disadvantage is that this methodology can be computationally intensive. This can be addressed by employing a variety of efficiency enhancements.

Two methods of Monte Carlo simulation were used for VaR measurement: Monte Carlo using covariance matrix and Monte Carlo using fitted models. The major advantage of the latter one is that it does not use assumption of normally distributed risk factors.

7. Solution of the optimization task

7.1 Target setting and assumption

The basis towards making investment according to the modern portfolio formation theory is H. Markowitz work. Taking into account mostly investment-oriented nature of the Sberbank portfolio of bonds, Markowitz optimization model could be used as a ground for conducting a further research. According to Markowitz there is no need to review all possible portfolios, but only those providing maximum expected return given the level of risk and minimum risk given the expected return. Consequently the optimization task is narrowed down to finding the possible portfolios frontier\(^3\) and positioning of the current Sberbank portfolio on the risk-expected return plain.

The major barrier consisted in the fact that original Markowitz problem was solved for one-factor financial instruments – stocks, i.e. only one risk factor was used in determination of risk estimate, while observed bonds were influenced by at least 10 risk factors – interest rates. In order to calculate risks for each instrument and portfolio in the whole it was necessary to pick risk factor groups, peculiar to the particular instrument, out of analyzed risk factor set. Setting each group of instrument certain weights could do this. For example today interest rate has wider range of instruments it influences, compared to 10-year interest rate. Having determined the weights, risks of the portfolio could be calculated with regard to instrument correlation. Such methodology has a right to live, but it was not adopted in the particular research for it assumed significant computation and development work. As a result the attempt to solve optimization model, where bonds were considered one-factor instruments was undertaken. Immediate yield was used as a risk factor. Accordingly its changes have been considered as risk. Logarithmic yield formula was adopted for yield calculation:

\[
Y = \ln \left( \frac{P_n}{P_{n-1}} \right)
\]

In such a way, the price was assumed to represent the described above risk factors\(^4\), i.e. the price incorporated risks brought by each risk factor. Substituting calculated values from the

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\(^3\) Efficient portfolio frontier according to Markowitz

\(^4\) Interest rates affecting the value of the bond
yield curve into the pricing function tested this hypothesis. The difference between the theoretical and actual market price was within the range allowed for standard statistical error (5\%).

7.2 Solution of the optimization task for Russian debt in USD

Optimization task was solved in the following way:

1. Statistical data was used to calculate covariance matrix of returns $COV_{ij}$ for all bonds consisting model portfolio. Markowitz model considers standard deviation from the expected return as risk. It is calculated as follows:

$$\sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot COV_{ij} \cdot x_j}$$

where

- $N$ - Number of the instruments in the portfolio,
- $x_i$ - weight of the $i$-th instrument in the portfolio,
- $COV_{ij}$ - element of the covariance matrix of the expected return time series of $i$-th and $j$-th instruments of the portfolio,
- $T$ - number of point in time series,
- $y_{it}$ - $t$-th value of the expected return time series of $i$-th instrument in the portfolio,
- $\bar{y}_i = \frac{1}{T} \sum_{t=1}^{T} y_{it}$ - mean of the expected return time series of $i$-th instrument in the portfolio.

2. The maximum and minimum mean current yields ($Y_{\min}$, $Y_{\max}$) for all instruments in the given time interval is calculated.

3. By specifying mean yields in the range between the maximum and minimum possible yields the structure of an optimal portfolio can be calculated to reduce risk. It means that for each given mean yield with increment lag of $\Delta Y_p = \frac{Y_{\max} - Y_{\min}}{K}$, $k$ - the number of partition points) the following steps are taken:

Finding portfolio weights ($x_i$) for each instrument,

in order to achieve minimum risk (min $\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot COV_{ij} \cdot x_j}$)

for a given mean yield of the portfolio ($Y_p = \sum_{i=1}^{N} x_i \bar{y}_i$)

and provided that the instrument will not be sold $\forall i$ $x_i \geq 0$),

while the total portfolio weights must be equal to 1 ($\sum_{i=1}^{N} x_i = 1$).
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As it was mentioned earlier, Sberbank portfolio of Russian debt instruments in US dollars has investment-oriented, thus assumption about constant structure of the portfolio, in particular case does not affect results of the research significantly.

Risk calculation formula has a quadratic form, thus a problem of achieving minimum risk for a given yield is a problem of quadratic programming. Moreover, introducing additional constraints can complicate the problem.

7.3 Solution of the problem using Value at Risk estimates

It is necessary to stress that initially attempts were made to draw efficient portfolios frontier based on the VaR-Expected return ratio, which is quite logical since VaR estimate calculated using Delta-Normal method and risk (according to Markowitz) are interrelated via fractile \( \alpha \):

\[ VaR = \alpha \cdot \delta \cdot \sqrt{n} \cdot PV \]

where:
- \( \alpha \) – fractile,
- \( \delta \) – standard deviation,
- \( n \) – time horizon
- \( PV \) – value of the portfolio.

In particular case construction of the efficient portfolio frontier would require minimization of risk calculation procedures, namely historical simulation, Monte Carlos, and RiskMetrics®, which is cost effective. Linear programming procedure within SAS/System used for such tasks could not be adapted to VaR models, since they are internal system procedures. SAS/Risk Dimensions has not provided possibility for implementation of linear programming tools available in other SAS applications, thus standard deviation value was used in construction of efficient portfolio frontier.
7.4 Results of the optimization task solution

The optimal portfolio set, obtained through the optimization task solution, is shown on Figure 1. Depending upon the model used for VaR calculation, positioning of the portfolio on the risk-expected return plain given the expected return was fluctuating within 0.2% of portfolio VaR. The VaR estimate obtained through the solution of the optimization task and the one calculated using the most comprehensive model – Monte Carlo provided the closest results.

![Optimal (efficient) portfolio frontier](image)

Figure 1. Optimal (efficient) portfolio frontier

During the solution of the task only one risk factor was used in VaR calculation – bond price on the market, while Monte Carlo method used several risk factors, however, the results of one factor and multi factor optimization model are close. This could mean that instrument price is valid and reflects interest rate change risks.

Except the graph, illustrating the current position of the Sberbank Eurobond portfolio, the structure of the optimal portfolio was then calculated. Based on the graph below conclusion regarding necessity of portfolio weights change aimed at profit maximization or risk minimization can be drawn. Apparently both figures show that the portfolio with minimum possible risk (0.8%) brings 7.1% of the expected return. The structure of this portfolio is shown on Figure 2. At the same time, it is obvious that investor could significantly increase profits by slight increase in risk. For example, with accepted overall risk of portfolio equal to 1%, investor is raising its profits to 10-11%.

The observed portfolio consists mostly of securities with maturity dates in 2008. Respectively securities maturing later, bring more risks as well as expected return in the portfolio, while at the same time, Russian eurobonds with maturing in 2003 are least risky and significantly reduce overall portfolio risk for their behavior is more or less predictable in short term. The graph shows that share of long-term debt (RF18, RF28) in any of possible portfolios is insignificant, unless revenues they promise and duration of 5-6 years. Such situation is similar
for majority of developing markets due to unpredictability of behavior of risk factors influencing the portfolio.

8. Research results

8.1 Results of VaR calculation

As a result of a conducted research VaR estimates for Russian debt portfolio nominated in US dollars were calculated using the following methods: RiskMetrics®, historical simulation, Monte Carlo, Monte Carlo using fitted models. Results of the calculation have shown that difference in VaR estimates evaluated using above models are insignificant, which proves sufficient depth of time series for conducting comprehensive statistical studies as well as rather stable tendencies prevailing on the market in the past years.

Furthermore standard Risk Dimensions tools provide possibility of VaR calculation given analytical section: markets, type of debt, particular security. Since risk is a nonadditive measure, its calculation is driven taking into account instrument and risk factor correlations. Results of VaR calculation by given analytical sections are illustrated below.
Figure 3. VaR calculation by eurobonds and Russian debt in USD

Figure 4. VaR calculation by instruments

8.2 Evaluation of model adequacy (back-testing)

Evaluation of models used for VaR calculation was performed on the resulting data. All models used in the research, satisfied requirements stated by Basel committee. Consequently, depending on the goal set before risk manager each of observed models can be used. The choice should be determined by specifics of the model behavior on particular time series, intervals peculiarities of the observed instruments.
8.3 Optimization task solution

During solution of the optimization task, which was based on the original Markowitz optimization model for stocks, possible (efficient) portfolios set and, at the same time, optimal portfolios set, represented by the possible (efficient) portfolios frontier were built. Therefore, it can be concluded that basic optimization models, originally developed for particular markets and instruments, can be successfully adopted for use with various financial assets (in particular eurobonds), however with certain restrictions imposed and assumptions made. During solution of the optimization task, additional assumption was made regarding representativity of eurobonds’ price, thus changes of the price were viewed as an aggregated index of risk factors, incorporating risk factors, which had been considered during the solution of multi-factor VaR models (Monte Carlo, historical simulation, RiskMetrics®).

8.4 Sensitivity analysis

Standard SAS/Risk dimensions tools allowed conduction of sensitivity analysis of each risk factor influencing overall portfolio risk when Monte Carlo techniques were used. Graph below shows that the most influence on the portfolio risk have 10 and 30-year benchmarks, while benchmark change affects the overall interest rates curve for Russian debt in US dollars. Setting more reference points for interest rate curve deformation would spread possible risks among them. Furthermore, the significant weight of 10-year benchmark among other risk factors can be described by the fact that most securities consisting the portfolio mature in 10 years.
8.5 Risk factor modeling
The ease of the input data change in Russian debt risk calculation project allowed vast possibilities for addition of various supplementary risk factors that could have influence on the overall portfolio risk and determining its value using sensitivity analysis.

8.6 Optimization task solution for the model stock portfolio
As a result of the above project development analogous task was solved for a model stock portfolio consisting of top-ten Russian stocks traded on MISEX. Portfolio risks were also calculated using four standard Risk Dimensions calculation techniques: RiskMetrics®, historical simulation, Monte Carlo, Monte Carlo using fitted models. Risk estimates were then used for in optimization task solution. In the particular case the original Markowitz model was reproduced.

8.7 SAS Risk Dimensions components utilization
Solutions developed with the help of SAS/Risk dimensions, have provided a wide range of different statistical and analytical applications except the one used for VaR calculation. During the project development multi dimensional warehouse for project data was constructed. Data obtained from various information sources, such as on-line quotes from Reuters, was cleared, processed, grouped and saved in accordance with predefined rules. Analytical instruments forming Risk Dimensions solution have provided possibilities for regression, cluster analysis, forecasting, neuron networks construction and so on.

Except using the statistical toolkit of Risk Dimensions, means for intranet application were development are also available and were used for creation of intranet applications, allowing interactive browsing of the driven research. Any changes inputted into the warehouse were immediately reflected at all stages of the solution hierarchy, including Intranet reports.
8.8 Portfolio modeling

As a consequence of the research, invested has been given a possibility to model different debt portfolios, including those, consisting foreign debt instruments, and, respectively, evaluate their influence on the portfolio risk.

8.9 International reporting

Risk management technologies, implemented in the optimization models solution, satisfy international requirements towards bank internal risk management models, established by Basel committee. Taking into consideration increasing attention of controlling and regulating organizations towards risk-oriented bank reporting and towards risk management methodic banks use, regular provision of risk-included reporting, could raise the bank’s authority on the market as well as its credit rating. Moreover, latest trends demonstrate adoption of Basel committee requirements by Russian regulating organizations, in particular the modern regulation on investment companies have taken a lot from there.

9. Bibliography