Interfacing a General Factoring Program with SAS: PROC MUFACT

Richard J. Hofmann and Joseph C. Simpson
Miami University*

Part I Introduction

The MUFACT procedure performs factor analyses and component analyses. The user may choose from a number of initial factoring methods, several transformation methods, and several ways of specifying SAS data set output files.

MUFACT may be applied to an ordinary SAS data set (containing raw data), a correlation matrix or a factor matrix. If the procedure is applied to raw data, MUFACT will exclude completely an observation having a missing value for a variable in the analysis. If the user wants to build the correlation matrix differently, he can (for example) use the COR procedure and submit the results to MUFACT. Memory available is the only limit to the number of variables that MUFACT will analyze.

MUFACT will operate with both singular and nonsingular data sets as well as with small and large samples. Defaults are set internally for singular matrices and small data sets. Second order factor analyses are supported. External communality estimate is supported insofar as the input data are rescaled prior to using MUFACT.

Part II Output Briefs

Partial and Multiple Correlations

Guttman (1953) has stated that one of the fundamental requirements of common factor analysis is that a representative psychometric sample be utilized in the analysis. Psychometric sampling refers to the sample of variables as opposed to subjects. Specifically Guttman states that for a sample of N variables the (N-2)th partial correlation between any pair of variables should approach zero. The ijth off-diagonal entry of this matrix represents the correlation between variables i and j with the effects of the other (N-2) variables partialed out.

In the diagonal of this matrix is the multiple correlation. The square of this multiple correlation represents the proportion of the variance of the variable that can be predicted by a linear regression equation utilizing the other (N-1) variables as predictors. The squared mutliple correlation is also a lower bound to the reliability of a variable.

Measuring Variable and Total Sampling Adequacy

These indices (MSA) are based upon Guttman's (1953) previously noted assumption for common factor analysis. The indices were developed by Kaiser (1970) and represent the efficiency with which the variable sample has been selected with regard to the other variables. We note how well a variable fits in with the other variables and how well the total composite of variables represent a psychometric sample.

Kaiser and Rice (1974) and Dziuban and Shirkey (1976) suggest that this value should be greater than .50 in order for the data to be acceptable for factor analysis. The index assumes a maximum value of unity. This index has generated some controversy and should not be accepted in all instances without question as it can be demonstrated that for certain types of correlation matrices the MSA indices will be lower than .50 even with good factor recovery.

Initial Factor Matrix

Interestingly the initial factor matrix is mainly of historical interest as it is usually rotated or transformed immediately after it has been obtained. One of the major problems in factor analysis is determining the number of factors. The factor analytic technique name image analysis, alpha factor anlaysis, components analysis (complete or incomplete) or whittle factor analysis is theoretically indicative of the factor determining technique.

Alpha factor analysis (Kaiser & Caffrey, 1964) is concerned with psychometric sampling. The alpha factors in the initial factor matrix have positive coefficient alpha's with regard to the theoretical psychometric universe. This property however, is destroyed by any transformation.

Complete components analysis (Hotelling, 1933) determines components in a hierarchical fashion. The first component explains as much of the intercorrelations, or variance, as is possible with a single component. The second component has a correlation of zero with the first component and explains as much of the remaining variance as is possible. Each component is determined such that it has an intercorrelation of zero with all other components--yet explains or accounts for as much of the intercorrelations remaining correlations as is possible. Components are determined until all of the variance has been accounted for. Unless one has linear dependency, a variable with a multiple correlation of unity--a singular matrix, there will be as many components as there are original variables. Program MUFACT checks for linear dependence and prints out a full page message if it exists.

The incomplete model is a hybridization of the complete components model. Probably a best reading source would be Gorsuch (1974). There are many ways to determine an "interpretable number: of components. In MUFACT the incomplete model stipulates that two criteria must be met: all eigenvalues greater than unity are retained (Guttman's, 1954, upper bound); at least 75 percent of the variance must be explained by the retained eigenvalues (rule of thumb). This dual

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"criterion" will usually determine a different number of components than either criterion will separately. As it turns out this criterion frequency yields results very similar to Cattell's (1966) screentest.

Image analysis (Guttman, 1953; Harris, 1962) is presently recognized as one of the most prominent of all factoring methods (even though it is not a factor analysis strictly speaking). It is rich in theory (see Gorsuch, 1974; Muliak, 1972) and more robust than factor analysis. When all of the underlying assumptions of factor analysis have been met, image analysis will yield results similar to those of factor analysis. When violations of the assumptions of factor analysis have occurred then image analysis will usually result in a better solution than that determined by the common factor model. Considerable unpublished work is presently being done with image analysis that suggests a robust image variation may soon be in the making for small sample analyses (a traditional problem for factor analysis). In theory image analysis defines factors having maximum canonical correlations with the psychometric universe variables (actually it is the variable composites that the factors define that have these maximum canonical correlations). As with the other factoring methods these properties are lost when the initial factor matrix is rotated or transformed.

Whittle (1953) factor analysis is a variation of the components model. Although the Whittle model has been around for some time many of its properties have only recently been recognized. Pruzek (1977) has pointed out a number of advantages associated with the Whittle model. In particular Pruzek has demonstrated that Whittle factors are very similar to maximum likelihood factors in approximating known population structures. Yet the Whittle model does not suffer from the problem of negative uniqueness estimates, a consistant problem of the maximum likelihood model.

Also included in MUFACT is a technique which we refer to as Diagonal. Frequently one reads of a method whereby communality estimates are placed in the diagonal prior to a components analysis. Such a procedure fails to adjust the off-diagonal correlations which frequently leads to a non-Gramian correlation matrix. In subroutine diagonal the off-diagonal correlations are rescaled by the communality estimates. This rescaling adjusts the correlations such that the matrix always remains Gramian. An incomplete components analysis is then used to analyze the rescaled correlation matrix.

Tests of Significance

Two tests of significance are included in MUFACT: testing the correlations matrix for significance and testing for the number of factors. Both tests are based upon Bartlett's (1950) chi square tests and my be applied with all factoring procedures.

In testing the correlation matrix for significance one is essentially testing to see if the correlation matrix is significantly different from an identity matrix.

Both of these significance tests generate large degrees of freedom, being a function of sample size, number of variables and number of factors, and tend not to be functional with large samples.

Communality Estimates

The communality estimate for each variable represents the proportion of variance for the variable that can be predicted or "explained" with-in the context of the factor solution. This communality estimate will not change when a factor matrix is rotated. It is a low estimate of the variables' reliability within the context of the factor solution. Variables with low communalities are not well explained by the factor solution and usually have low coefficients in the solution matrix. However, such variables may be measuring something quite unlike the other variables in the analysis and for that reason alone it should be worthy of considerable investigation.

Orthogonal Transformation Solution (Rotated Solution)

This solution matrix is determined through a maximization of the orthomax criterion with some weight, say gamma. Depending upon one's choice for gamma a number of different solutions may be obtained, e.g., varimax, quartimax, equamax (see Gorsuch, 1974 or Mulaik, 1972).

With an orthogonal transformation solution some semblance of simple structure, high or zero loadings is obtained and the factors are independent or uncorrelated. Usually a quartimax solution will load a majority of the variables on the first few factors at the expense of some latter factors, i.e., most of the proportionate contributions of the factors will be explained by the first several factors. The equamax solution tends to "spread" the variance contributions across all of the factors. The varimax solution is a "happy" medium between the quartimax and equamax.

There seems to be "theoretical unrest" in the discussions of which of the three solutions is the "best". Some factor analysts swear by the varimax while other analysts swear by the equamax. When one has a two factor solution the varimax and equamax solutions are identical. Even with a many factor solution it's possible to get varimax and equamax solutions that are almost identical. At other times there is very little similarity between the two solutions.

On many occasions it has been noted that those variables with large communalities seem to have good simple structure, high or zero loadings, while those variables with low communalities seem to have poor simple structure. Alternatively the greater a variable's communality the more influence it exerts on the transformation solution. To overcome this problem, Kaiser (1958) has suggested that all variables be weighted during the transformation computations so that each has the same communality (normalized). When we weight the variables accordingly we say the solution was computed from a normalized matrix or it is a normalized matrix or it is a normal solution as opposed to a "raw" solution. The variables are "unweighted" (denormalized) after the transformation process.

The loadings in an orthogonal transformation solution are the correlations of the variables with the factors. They show the relationship of a factor to a variable. Interestingly enough they provide a measure of the independent contribution each factor makes to the variance of the variables and are in a very special sense standard regression coefficients of the variables on the factors.

Oblique Solution (Primary Pattern Matrix) or Oblique Solution (Reference Structure Matrix)

The oblique solution is the general transformation solution and admits to an orthogonal solution as a special case. That is, in general we seek to find a best simple structure representation for a given group of variables and factors regardless of the factor intercorrelations (recall in the orthogonal solution the factors are uncorrelated). If the best representation for the factors is an orthogonal representation then the computing algorithm used (Hofmann's, 1976 orthotran) will define an orthogonal solution. If on the other hand the best simple structure representation for the factors is an oblique representation then the computing procedure (the orthotran) will define an oblique solution.

There are two types of oblique solution matrices used in factor analysis either one of the two is usually reported as a solution but typically both are not reported. The primary pattern is computed in the spirit of Holzinger and Harman (1941) and Harman (1976) while the reference structure is computed in the spirit of Thurstone (1947). Actually one is easily determined from the other (see Harris and Knoell, 1948). Both solutions have been shown to be derived from the same basic matrix (Hofmann, 1976) so the user may use whichever solution is the most comfortable to interpret.

Primary Pattern

The pattern loadings or coefficients may be interpreted as measures of the independent contribution each factor makes to the variance of the variables. They measure the dependence of the variables on the different factors and in this sense they are regression coefficients of the variables on the factors. If a factor-axis is placed through a cluster of variables the pattern coefficients for the variables within the cluster will be zero on all other factors and they will be relatively substantial on the factor whose axis passes through the cluster. Sometimes they achieve magnitudes slightly greater than unity (see Rummel, 1970), even though no computational error has been made.

Reference Structure

The reference structure loadings may be interpreted as correlations. The loading of a variable on a factor is the correlation of the variable with the factor with the effects of the other factors partialled out. It reflects the distinct relationship of the factor to the variable, a relationship which is statistically independent of any of the other factors in the analysis. Inasmuch as the loadings are correlations we talk of unit loadings, correlations of one, as opposed to the zero coefficients of a primary solution. (see Gorsuch, 1974).

If one is doing an exploratory factor analysis it might well be appropriate to look at all three solutions although Cattell (1966) along with a number of other factor analysts feels that one should avoid using an orthogonal solution when oblique solutions are available.

In reading or reporting research results, it is necessary to know exactly which matrix is presented as a factorial solution. Because of the overwhelming influence of Thurstone many older studies report a reference structure solution.

Intercorrelations of the Primary Factors

The intercorrelations of the primary factors is just an intercorrelations matrix. If one has an orthogonal factor matrix the factor intercorrelations are implicitly zero and are not reported. However if the factors are correlated, even a "smidgeon", then their intercorrelations are reported.

Very seldom will a factor intercorrelation be greater than .40 in magnitude. When the factor intercorrelations tend to be above .40 it is usually hypothesized that there is a second order factor structure accounting for the intercorrelations of the variables. In such a situation the researcher might profit from a second order factor analysis, which is just a factor analysis of the intercorrelations of the primary factors (see Gorsuch, 1974 for further discussion of higher-order factors). The program is capable of performing second order or higher order analyses.

Direct and Joint Proportionate Contributions

Many researchers using a components model have a proclivity to refer to "the amount of variance accounted for". In particular it may be desirable to make certain judgments or decisions about a factor on the basis of its contributions. Typically one does not know without great difficulty what the total variance of a "factored matrix" really is. However we can determine the common variance, the amount of variance associated with the common factors. In most situations it is of interest to know what proportion of the common variance may be explained by one factor independently of the other factors, a direct contribution, or that proportion of the common variance that may be explained by the intercorrelation of one factor with the other factors, a joint contribution (which is usually very small and sometimes negative) or that proportion of the common variance that may be explained by both joint and direct contributions, the total contribution of a factor (see Hofmann, 1975-b).

Variable Complexity

Frequently one likes to know whether a variable is factorially complex (requiring many factors to describe it) or factorially simple (requiring few factors to describe it). The variable complexity is nothing more than an index of the number of factors required to describe a variable (Hofmann, 1976-a). The independent cluster solution for instance defines a factor solution in which all variables have a factor complexity of unity.

The average complexity is simply the average number of factors describing a group of variables. Alternatively it represents the average number of factors each variable "loads" on.

A solution having a number of variables with relatively large complexities (say anything over 2.5) is usually a poor solution. Such a solution may be: (a) underfactored; (b) defined by an improper transformation solution; (c) composed of a group of heterogeneous variables which should never have been factor analyzed to start with; (d) highly unstable or non-generalizable. When you have relatively large complexities either abandon the solution or seek expert advice as to why the variables are complex.

Regression Estimate Factor Scores

On many occasions a researcher will know the theoretical scores obtained by his sample on the factors. That is, assuming a factor to be a hypothetical variable the hypothetical score we expect from a given sampling entity on this hypothetical variable is a factor score. While techniques are a "dime-a-dozen" for determining factor scores we have developed a new variation (Hofmann, 1975-a) for use in this program. This new technique is really just a variation of an old technique (Thurstone, 1935) that utilizes a multiple regression approach to estimate the factor scores. In particular this approach generalizes to either orthogonal or oblique solutions: small samples or large samples; singular and non-singular matrices.

Factor Score Weight Matrix

Once a factor solution has been determined it is sometimes desirable to retain the weight matrix for determining the factor score. If you have sampling entities not included in an analysis but you wish to estimate their factor score weight matrix provides the necessary weights.

Factor Scores

• The factor scores are estimated in standard score form, mean of zero and standard deviation of unity for each factor. The intercorrelations of the factor scores should be approximately the same as the primary intercorrelation matrix unless one has computed an orthogonal solution, in which case the intercorrelation of the factors will be zero.

A reflection routine has been placed in the program. This routine reflects the factor scores so that they show the same directionality as the factors in the solution matrix. Thus, the intercorrelation of the factor scores should, within rounding errors, be identical to the factor intercorrelation matrix.

Part III General Output

1. means, standard deviations, coefficients of variation
2. (n-2) order partial correlations and squared multiple correlations
3. initial factor matrix
4. proportionate contributions of the factors
5. communality estimates
6. chisquare tests (optional)
7. orthogonal rotation
8. proportionate contributions of orthogonal factors
9. variable complexity on orthogonal factors
10. orthonormal transformation matrix (optional output)
11. eigenvalues and eigenvectors (optional output)
12. oblique primary pattern
13. oblique reference structure
14. oblique primary intercorrelation
15. oblique primary structure (optional output)
16. variable complexity on oblique factors
17. proportionate contributions of oblique factors
18. measure of sampling adequacy (a) variable (b) total
19. psychometric sampling adequacy of oblique factors (optional output)
20. (r-2) order partial correlations and squared multiples of factors (optional output)
21. factor score weight matrix
22. factor scores for individuals - Printed or SAS Data file

The Procedure MUFACT Statement

Options and Parameters

METHOD =	IMAGE	a Harris (1963) type image analysis
	ALPHA	a Kaiser and Caffry (1964) alpha factor analysis
	COMPONENT	a traditional Hotelling (1933) components analysis
	DIAGONAL	the squared mutliple correlations are placed in the diagonal and the off
	WHITTLE	a Whittle analysis
Default is IMAGE		
FACTORS		<u>a priori</u> number of factors. If you are entering a factor matrix this will index the number of columns. If your estimate will create a computational breakdown it will be overridden by the method. (optional) The number will be determined by factoring method.
	QMAXORT	orthogonal quartimax
	VMAXORT	orthogonal varimax
	EMAXORT	orthogonal equamax
	OMAXORT	some nontraditional orthogonal solution
	QMAXOBL	oblique analog quartimax
	VMAXOBL	oblique analog varimax

	EMAXOBL	oblique analog equamax
Default		no transformation
GAMMA =		some orthomax weight - do not use this unless you fully understand the orthomax criterion (see Mulaik, 1972)
Default		set ROTATE
INPUT =	COR	correlation matrix punched in lower left triangular form, with diagonal included -- row by row
	FACTOR	a factor matrix is entered row by row
	RAW	raw data are entered with each record card representing the scores for a single subject
Default		raw data
SCORES =	PRINT	print the factor scores subject by subject
	SASOUT	write the factor scores subject by subject on a SAS data file -- no printing of scores (OUT must be specified)
	BOTH	print factor scores and store them on SAS data file (out must be specified)
Default		no factor scores are computed
FACTOUT =	SAS-DATA-Set-NAME (Factor matrix output)	
OUT =	SAS	data set name for factor scores
ITER =		maximum number of interactions for alpha analysis limits 1-1000. Default 30
DATA =	SAS	data set name - <u>optional</u>
RAW		option to compute raw transformation solutions
RAW2		option to compute raw second order transformation solutions
EXTRA		option for extra output
SECOND		option for second order factor analysis (first order must be oblique)
CHISQ		performs Chi square tests of significance for correlation matrix and number of factors
SECONDM =		identical to method--defines method of analysis for second order solution
<u>Default</u>	IMAGE	
SECONDR =		method of second order transformation identical to ROTATE and GAMMA
<u>Default</u>		orthotran
SECONDG =		this is identical to GAMMA and should not be used unless you understand the orthomax criterion (see Mulaik, 1973)
<u>Default</u>		set by SECONDR
SECONDF =		*a priori* number of second order factors to be extracted (optional)

NOTES

PROC MUFACT	will result in a blind image analysis with an orthotran transformation and no extra output
MUFACT	will not create an output data set of factor scores if a BY statement is used
MUFACT	in an expanded form, will be available in the supplementary procedures library

Part IV Interfacing

The MUFACT program was first installed in SAS72, then converted to SAS76. In the opinion of this paper's authors, the Institute supplied programs were of insufficient scope for many factor analyses. It would be unreasonable to expect the Institute to upgrade PROC FACTOR in the near term because their staff is both small and (in the area of factor analysis) inexperienced. Thus we decided that addition of this procedure would materially aid SAS users who require a factor analysis tool. We say tool, because that is what many of the SAS procedures are. They accept input in a standard way and transform it to produce output in a standard format acceptable to other procedures (Many analyses are the result of stringing together several tools (procedures) in a pipeline, each tool transforming the data in some way. This cooperative relationship between SAS procedures greatly magnifies the benefits of installing any SAS procedure. For an excellent discussion of design criteria and use of programs as tools see Kernigham and Plauger (1976).

Given a sufficient return for improving factor analytic abilities the question of how to implement them arises. There are generally three options, use existing PROCS and data manipulation facilities to construct one or more MACRO's, use PROC MATRIX, or install a new procedure. These options are listed roughly in order of increasing effort of implementation and increasing ease of use. In particular, procs can handle options more effectively and can issue much better diagnostics.

Once it was decided to install PROC MUFACT, the installation naturally divided itself into two tasks: 1) specify/modify the stand alone FORTRAN program to be an effective tool in the SAS environment, and 2) provide the interface with the SAS supervisor.

The effort involved in the first task came as quite a surprise. The stand-alone version of the program had many options to accept input from differing sources. As expected, the deletion/replacement of this code was quite easy. However, this program also contained complex logic to control the sequence of calculation. The program was not a simple tool, but a complex problem solving machine. Expunging this code and installing the good error checking code required of a tool consumed much more effort than anticipated (an estimated 160 man hours). Most of this effort was incurred by R. Hofmann because J. Simpson did not realize the importance of communicating the tool concept.

Providing the interface with the SAS supervisor could also be decomposed into sub tasks:

1) obtain control information and data, and 2) create a SAS output data set. The first sub-task required only a few hours effort. It was greatly simplified by the powerful and easily invoked features of SAS 76 parseing modules, in particular, the facile PARM checking and ability to control the type of variables in a list. Errors caught in the language module use the standard SAS convention of underlining the offending information and printing a terse error message.

The second sub-task, providing for creation of a SAS data set of factor scores, required approximately 20 man hours, ten hours of coding and ten hours to design the solution. While the solution was quite simple, abstracting it from the documentation was not.

Summarizing, the effort of installing MUFACT was substantial. Much of this effort could have been avoided if a clear idea of the structure of a SAS proc had been stated at project inception.

The inclusion of, and error checking, for options, parms, and variable lists is quite easy and does not require special skills. Creation of a SAS output data set was made more difficult because the documentation was not clear on this point.

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1                                       S T A T I S T I C A L   A N A L Y S I S   S
NOTE: THE JOB ABMMU647 HAS BEEN RUN UNDER RELEASE 76.2 OF SAS AT SOUTHWEST OHIO RE(

1          MACRO HOFTEST MUFACT CHISQ  SCORES=PRINT%
2          TITLE TEST OF PROC MUFACT;
3          TITLE3 THURSTONE BOX PROBLEM;
4          DATA HELPER;
5          INPUT V1 1-11 V2 12-22 V3 23-33 V4 34-44 V5 45-55
6          V6 56-66 V7 67-77
7          / V8  1-11 V9  12-22 V10  23-33 V11  34-44 V12  45-55 V13  56-66
8          V14  67-77
9          / V15  1-11 V16  12-22 V17  23-33 V18  34-44 V19  45-55 V20  56-66;
10         LABEL
11         V1  = X
12         V2  = Y
13         V3  = Z
14         V4  = XY
15         V5  = XZ
16         V6  = YZ
17         V7 =  SQRT(X*X + Y*Y)
18         V8=  SQRT(X*X + Z*Z)
19         V9 =  SQRT(Y*Y + Z*Z)
20         V10 = 2X + 2Y
21         V11 = 2X + 2Z
22         V12 = 2Y + 2Z
23         V13 = LOG(X)
24         V14 = LOG(Y)
25         V15 = LOG(Z)
26         V16 = XYZ
27         V17 = SQRT(X*X + Y*Y + Z*Z)
28         V18 = EXP(X)
29         V19 = EXP(Y)
30         V20 = EXP(Z);
31         CARDS;

NOTE: DATA SET WORK.HELPER HAS 20 OBSERVATIONS AND 20 VARIABLES.
NOTE: THE DATA STATEMENT USED 0.15 SECONDS AND 96K.

92         PROC PRINT;
93         PROC HOFTEST

NOTE: THE PROCEDURE PRINT USED 0.16 SECONDS AND 128K AND PRINTED PAGE 1.

93                             METHOD=COMP ROTATE=VMAXOBL GAMMA=1;
94         PROC HOFTEST

NOTE: THE PROCEDURE MUFACT USED 1.55 SECONDS AND 192K AND PRINTED PAGES 2 TO 14.

94                             METHOD=IMAGE;
95         TITLE TEST OF HOFMANNS MUFACT PROGRAM;


                                                  TEST OF PROC MUFACT

                                                THURSTONE BOX PROBLEM
125K BYTES REQUIRED FOR THIS ANALYSIS ASSUMING A NON-SINGULAR CORRELATION MATRIX
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TEST OF PROC MUFACT 0:58 FRIDAY,

THURSTONE BOX PROBLEM

VARIABLE	MEAN	VARIANCE	STAN. DEV.	SKEWNESS	KURTOSIS	
1 V1	17.40	40.67	6.38	0.02	-1.38	X
2 V2	9.60	22.99	4.79	0.25	-1.33	Y
3 V3	4.20	9.85	3.14	0.56	-1.07	Z
4 V4	12.45	20.68	4.55	0.34	-1.05	XY
5 V5	7.85	14.03	3.75	0.46	-0.80	XZ
6 V6	5.85	10.13	3.18	0.55	-0.82	YZ
7 V7	5.12	0.78	0.88	-0.17	-1.01	SQRT(X*X + Y*Y)
8 V8	4.58	0.67	0.82	-0.19	-0.96	SQRT(X*X + Z*Z)
9 V9	3.61	0.79	0.89	-0.14	-1.12	SQRT(Y*Y + Z*Z)
10 V10	14.20	6.27	2.50	-0.03	-1.00	2X + 2Y
11 V11	12.00	5.47	2.34	0.0	-0.81	2X + 2Z
12 V12	9.80	6.27	2.50	0.03	-1.00	2Y + 2Z
13 V13	0.60	0.01	0.09	-0.37	-1.18	LOG(X)
14 V14	0.46	0.01	0.12	-0.26	-1.33	LOG(Y)
15 V15	0.24	0.04	0.19	-0.20	-1.47	LOG(Z)
16 V16	24.45	226.47	15.05	0.79	-0.28	XYZ
17 V17	5.51	0.85	0.92	-0.19	-0.88	SQRT(X*X + Y*Y + Z*Z)
18 V18	78.81	2939.20	54.21	0.42	-1.55	EXP(X)
19 V19	26.63	381.96	19.54	0.62	-1.29	EXP(Y)
20 V20	8.93	47.96	6.93	0.85	-0.88	EXP(Z)

NUMBER OF ENTITIES = 20.

RELATION MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1.000															
2	0.262	1.000														
3	0.098	0.247	1.000													
4	0.669	0.885	0.248	1.000												
5	0.487	0.304	0.894	0.477	1.000											
6	0.190	0.606	0.904	0.561	0.859	1.000										
7	0.859	0.712	0.207	0.942	0.515	0.459	1.000									
8	0.905	0.339	0.504	0.686	0.789	0.545	0.838	1.000								
9	0.231	0.878	0.668	0.787	0.657	0.887	0.632	0.488	1.000							
10	0.779	0.805	0.236	0.980	0.508	0.519	0.988	0.780	0.720	1.000						
11	0.741	0.338	0.731	0.613	0.937	0.735	0.719	0.951	0.602	0.683	1.000					
12	0.216	0.773	0.795	0.701	0.771	0.960	0.567	0.527	0.979	0.644	0.683	1.000				
13	0.987	0.288	0.097	0.681	0.483	0.199	0.872	0.903	0.250	0.795	0.739	0.231	1.000			
14	0.213	0.978	0.299	0.853	0.329	0.635	0.672	0.317	0.902	0.773	0.338	0.805	0.237	1.000		
15	0.104	0.198	0.949	0.213	0.891	0.864	0.185	0.493	0.620	0.206	0.734	0.763	0.101	0.246	1.000	
16	0.459	0.626	0.824	0.710	0.917	0.947	0.658	0.734	0.857	0.693	0.861	0.916	0.462	0.634	0.787	1.000
17	0.794	0.713	0.487	0.915	0.725	0.681	0.954	0.900	0.770	0.953	0.862	0.749	0.804	0.693	0.456	0.829
18	0.980	0.220	0.097	0.629	0.476	0.171	0.813	0.875	0.200	0.732	0.717	0.191	0.937	0.175	0.105	0.438
19	0.295	0.984	0.194	0.881	0.273	0.560	0.722	0.346	0.827	0.804	0.326	0.719	0.322	0.924	0.151	0.558
20	0.093	0.260	0.991	0.255	0.867	0.893	0.210	0.493	0.667	0.241	0.707	0.785	0.092	0.312	0.898	0.814
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

	17	18	19	20
17	1.000			
18	0.753	1.000		
19	0.704	0.250	1.000	
20	0.485	0.090	0.206	1.000
	17	18	19	20

```
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***                  ***
*** SINGULAR MATRIX  ***
***                  ***
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```

80 INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

	EIGENVALUE	ACCCUNTABLE VARIANCE	CUMMULATIVE VARIANCE
1	12.61493	0.6307464	0.6307464
2	4.03697	0.2018485	0.8325949
3	2.98140	0.1490698	0.9816647
******************ONLY THE ABOVE WILL BE USED IN FURTHER ANALYSES****			
4	0.11556	0.0057782	0.9874429
5	0.09122	0.0045609	0.9920038
6	0.07508	0.0037538	0.9957576
7	0.05379	0.0026896	0.9984473
8	0.02019	0.0010094	0.9994567
9	0.01050	0.0005248	0.9999815
10	0.00029	0.0000144	0.9999959

INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

COMMUNALITY ESTIMATES

		1
V1	1	0.955
V2	2	0.999
V3	3	0.990
V4	4	0.950
V5	5	0.982
V6	6	0.985
V7	7	0.996
V8	8	0.992
V9	9	0.990
V10	10	0.998
V11	11	0.996
V12	12	0.996
V13	13	0.982
V14	14	0.974
V15	15	0.946
V16	16	0.963
V17	17	0.993
V18	18	0.954
V19	19	0.959
V20	20	0.950

INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

OBLIQUE PRIMARY PATTERN SOLUTION MATRIX(ORTHOTRAN)

		1	2	3
V1	1	1.001	-0.008	-0.003
V2	2	0.019	0.032	0.987
V3	3	-0.009	0.997	-0.005
V4	4	0.482	0.036	0.749
V5	5	0.399	0.869	-0.023
V6	6	0.004	0.826	0.394
V7	7	0.734	0.017	0.513
V8	8	0.860	0.408	0.006
V9	9	-0.006	0.506	0.753
V10	10	0.622	0.033	0.633
V11	11	0.672	0.668	-0.011
V12	12	-0.003	0.671	0.605
V13	13	0.585	-0.014	0.031
V14	14	-0.023	0.093	0.969
V15	15	0.013	0.982	-0.056
V16	16	0.290	0.726	0.364
V17	17	0.649	0.316	0.460
V18	18	0.987	-0.000	-0.044
V19	19	0.062	-0.021	0.967
V20	20	-0.018	0.973	0.017

1

CHI-SQUARE TEST FOR NUMBER OF FACTORS IS 0.5813274D 02
WITH 133 DEGREES OF FREEDOM THE CHANCE PROBABILITY OF OBTAINING THIS VALUE OR SOME LARGER VALUE IS 0.0000
NOTE THAT IT IS MOST DESIRABLE TO HAVE A NONSIGNIFICANT CHI-SQUARE. (Note test is only appropriate for large samples)

VARIABLE COMPLEXITY(APPROXIMATE NUMBER OF FACTORS THAT A VARIABLE LOADS ON

		1			
V1	1	1.000	V11	11	2.000
V2	2	1.003	V12	12	1.979
V3	3	1.000	V13	13	1.002
V4	4	1.712	V14	14	1.021
V5	5	1.405	V15	15	1.007
V6	6	1.433	V16	16	1.827
V7	7	1.791	V17	17	2.313
V8	8	1.428	V18	18	1.004
V9	9	1.749	V19	19	1.009
V10	10	2.005	V20	20	1.001

1

AVERAGE VARIABLE COMPLEXITY = 1.434

INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

OBLIQUE REFERENCE STRUCTURE SOLUTION

		1	2	3
V1	1	0.968	−0.008	−0.003
V2	2	0.018	0.031	0.936
V3	3	−0.009	0.970	−0.005
V4	4	0.466	0.035	0.711
V5	5	0.385	0.845	−0.022
V6	6	0.004	0.804	0.374
V7	7	0.710	0.017	0.487
V8	8	0.832	0.397	0.006
V9	9	−0.006	0.492	0.715
V10	10	0.602	0.032	0.601
V11	11	0.650	0.649	−0.011
V12	12	−0.003	0.652	0.574
V13	13	0.952	−0.014	0.029
V14	14	−0.032	0.091	0.920
V15	15	0.012	0.955	−0.053
V16	16	0.280	0.706	0.345
V17	17	0.628	0.307	0.437
V18	18	0.954	−0.000	−0.042
V19	19	0.060	−0.020	0.918
V20	20	−0.017	0.946	0.016

INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

FACTOR SCORE WEIGHT MATRIX

		1	2	3
V1	1	−0.189	−0.037	−0.043
V2	2	0.039	−0.036	0.201
V3	3	0.037	0.178	−0.043
V4	4	−0.058	−0.042	0.132
V5	5	−0.045	0.142	−0.058
V6	6	0.046	0.131	0.046
V7	7	−0.116	−0.045	0.074
V8	8	−0.147	0.041	−0.052
V9	9	0.051	0.059	0.134
V10	10	−0.089	−0.043	0.102
V11	11	−0.103	0.095	−0.059
V12	12	0.050	0.094	0.096
V13	13	−0.184	−0.039	−0.035
V14	14	0.051	−0.023	0.197
V15	15	0.031	0.177	−0.053
V16	16	−0.013	0.104	0.032
V17	17	−0.091	0.014	0.054
V18	18	−0.187	−0.034	−0.051
V19	19	0.028	−0.047	0.197
V20	20	0.039	0.174	−0.037

A GENERALIZED INVERSE WAS USED.

INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

DIRECT AND JOINT PROPORTIONATE CONTRIBUTIONS OF FACTORS TO TOT. COMMON VARIANCE

	DIRECT	+	JOINT	=	TOTAL
1	(0.311)	+	(−0.002)	=	0.310
2	(0.332)	+	(0.084)	=	0.415
3	(0.277)	+	(−0.002)	=	0.275

ORTHOMAX CRITERION IS = 1.00
WITH A NORMALIZED MATRIX

INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

REGRESSION ESTIMATE FACTOR SCORES

1	1.3688	−1.0464	−1.1585
2	1.3353	−0.0243	−1.2019
3	1.3538	−1.0374	−0.0378
4	1.3285	0.0378	−0.0857
5	1.3117	1.3389	−0.1418
6	0.2094	−1.0705	−1.1489
7	0.1857	−0.0006	−1.2014
8	0.2052	−1.0579	−0.0121
9	0.1868	0.0704	−0.0658
10	0.1811	1.4235	−0.1275
11	0.2000	−1.1051	1.3682
12	0.1902	0.0852	1.3215
13	0.1933	1.4991	1.2638
14	−1.1689	−1.1255	−1.1909
15	−1.1900	−0.0119	−1.2535
16	−1.1859	0.0608	−0.0953
17	−1.1861	1.4735	−0.1640
18	−1.1678	−1.1628	1.3654
19	−1.1788	0.0893	1.3141
20	−1.1723	1.5639	1.2523

INCOMPLETE COMPONENTS ANALYSIS (HOTELLING)

INTERCORRELATIONS OF PRIMARY FACTORS

		1	2	3
FACTOR	1	1.000		
FACTOR	2	0.117	1.000	
FACTOR	3	0.247	0.223	1.000

```
NOTE: THE PROCEDURE MUFACT USED 0.23 SECONDS AND 192K AND PRINTED PAGES 15 TO 18.

          DATA PHYS8  (TYPE=CORR DF=304);
          TITLE 'EIGHT PHYSICAL VARIABLES'
          TITLE3 'HARMON, MODERN FACTOR ANALYSIS, 3ND ED.';
          * SEE PP. 124-125 OF HARMAN: MODERN FACTOR ANALYSIS, 2ND ED;
          INPUT _NAME_ $ 1-8 _TYPE_ $ 73-80
          VAR1 9-16 VAR2 17-24 VAR3 25-32 VAR4 33-40
          VAR5 41-48 VAR6 49-56 VAR7 57-64 VAR8 65-72;
          LABEL
          VAR1=HEIGHT VAR2=ARM SPAN VAR3=LENGTH OF FOREARM VAR4=LENGTH OF LOWER LEG
          VAR5=WEIGHT VAR6=BITROCHANTERIC DIAMETER VAR7=CHEST GIRTH VAR8=CHEST WIDTH;
          CARDS;

NOTE: DATA SET WORK.PHYS8 HAS 8 OBSERVATIONS AND 10 VARIABLES.
NOTE: THE DATA STATEMENT USED 0.06 SECONDS AND 96K.

          PROC PRINT;
          TITLE EIGHT PHYSICAL VARIABLES FACTORED FOUR WAYS;
          PROC HOFTEST

NOTE: THE PROCEDURE PRINT USED 0.11 SECONDS AND 116K AND PRINTED PAGE 19.

                      METHOD=IMAGE INPUT=COR ROTATE=VMAXOBL;
          TITLE EIGHT PHYSICAL VARIABLES;
          TITLE3 INPUT IS A CORRELATION MATRIX;
          TITLE5 IMAGE ANALYSIS FOLLOWED BY ORTHOTRAN SOLUTION;
          PROC HOFTEST

NOTE: THE PROCEDURE MUFACT USED 1.05 SECONDS AND 192K AND PRINTED PAGES 20 TO 31.

                      METHOD=ALPHA INPUT=COR ROTATE=VMAXOBL;
          PROC HOFTEST

NOTE: THE PROCEDURE MUFACT USED 1.32 SECONDS AND 192K AND PRINTED PAGES 32 TO 50.

                      METHOD=IMAGE INPUT=COR ROTATE=VMAXOBL;
          TITLE3 INPUT IS A CORRELATION MATRIX;
          TITLE5 IMAGE FACTOR EXTRACTION FOLLOWED BY ORTHOTRAN ROTATION;
          PROC HOFTEST

NOTE: THE PROCEDURE MUFACT USED 0.99 SECONDS AND 192K AND PRINTED PAGES 51 TO 62.

                      METHOD=DIAG INPUT=COR ROTATE=VMAXOBL;
          TITLE5 DIAGONAL FACTOR EXTRACTION FOLLOWED BY ORTHOTRAN ROTATION;
          PROC HOFTEST

NOTE: THE PROCEDURE MUFACT USED 1.00 SECONDS AND 192K AND PRINTED PAGES 63 TO 75.

                      METHOD=ALPHA  ROTATE=VMAXORT;
          TITLE5 ALPHA FACTOR EXTRACTION FOLLOWED BY ORTHOTRAN ROTATION;

NOTE: THE PROCEDURE MUFACT USED 0.80 SECONDS AND 192K AND PRINTED PAGES 76 TO 93.
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EIGHT PHYSICAL VARIABLES

INPUT IS A CORRELATION MATRIX

IMAGE ANALYSIS FOLLOWED BY ORTHOTRAN SOLUTION

121K BYTES REQUIRED FOR THIS ANALYSIS ASSUMING A NON-SINGULAR CORRELATION MATRIX

ORIGINAL CORRELATION MATRIX

		1	2	3	4	5	6	7	8
VAR1	1	1.000							
VAR2	2	0.846	1.000						
VAR3	3	0.805	0.881	1.000					
VAR4	4	0.859	0.826	0.801	1.000				
VAR5	5	0.473	0.376	0.380	0.436	1.000			
VAR6	6	0.398	0.326	0.319	0.329	0.762	1.000		
VAR7	7	0.301	0.277	0.237	0.327	0.730	0.583	1.000	
VAR8	8	0.382	0.415	0.345	0.365	0.629	0.577	0.539	1.000
		1	2	3	4	5	6	7	8

PARTIAL AND MULTIPLE CORRELATIONS

		1	2	3	4	5	6	7	8
VAR1	1	0.903							
VAR2	2	0.346	0.922						
VAR3	3	0.072	0.584	0.895					
VAR4	4	0.479	0.179	0.188	0.888				
VAR5	5	0.183	-0.196	0.100	0.056	0.865			
VAR6	6	0.103	-0.005	0.027	-0.122	0.492	0.777		
VAR7	7	-0.146	0.091	-0.116	0.131	0.491	0.054	0.750	
VAR8	8	-0.086	0.248	-0.087	-0.025	0.238	0.177	0.120	0.691
		1	2	3	4	5	6	7	8

VARIABLE SAMPLING EFFICIENCY

		1
VAR1	1	0.864
VAR2	2	0.816
VAR3	3	0.858
VAR4	4	0.667
VAR5	5	0.780
VAR6	6	0.851
VAR7	7	0.824
VAR8	8	0.838

1
TOTAL SAMPLING EFFICIENCY = 0.8455

INCOMPLETE IMAGE ANALYSIS (HARRIS)

COMMUNALITY ESTIMATES

		1
VAR1	1	0.941
VAR2	2	0.959
VAR3	3	0.907
VAR4	4	0.932
VAR5	5	0.918
VAR6	6	0.900
VAR7	7	0.896
VAR8	8	0.706

1

SIGNIFICANCE TEST IS NOT APPROPRIATE

AS SAMPLE MUST BE GREATER THAN

(VARIABLES-NUMBER OF FACTORS)/3 +2

INCOMPLETE IMAGE ANALYSIS (HARRIS)

OBLIQUE PRIMARY PATTERN SOLUTION MATRIX(ORTHOTRAN)

		1	2	3	4
VAR1	1	0.902	0.185	-0.061	-0.134
VAR2	2	0.934	-0.028	0.034	0.277
VAR3	3	0.912	0.072	-0.087	0.268
VAR4	4	0.915	-0.114	0.205	-0.173
VAR5	5	0.060	0.608	0.438	-0.001
VAR6	6	-0.010	0.954	-0.000	0.149
VAR7	7	-0.039	0.030	0.940	0.017
VAR8	8	0.085	0.327	0.452	0.435

INCOMPLETE IMAGE ANALYSIS (HARRIS)

VARIABLE COMPLEXITY(APPROXIMATE NUMBER

OF FACTORS THAT A VARIABLE LOADS ON

VAR1	1	1.139
VAR2	2	1.179
VAR3	3	1.205
VAR4	4	1.208
VAR5	5	1.841
VAR6	6	1.049
VAR7	7	1.006
VAR8	8	2.904

1
AVERAGE VARIABLE COMPLEXITY = 1.441

INCOMPLETE IMAGE ANALYSIS (HARRIS)

OBLIQUE REFERENCE STRUCTURE SOLUTION

		1	2	3	4
VAR1	1	0.836	0.146	-0.049	-0.133
VAR2	2	0.865	-0.022	0.027	0.275
VAR3	3	0.845	0.057	-0.070	0.266
VAR4	4	0.847	-0.090	0.164	-0.171
VAR5	5	0.055	0.479	0.352	-0.001
VAR6	6	-0.009	0.752	-0.000	0.148
VAR7	7	-0.036	0.024	0.754	0.017
VAR8	8	0.082	0.256	0.362	0.432
		1	2	3	4

INCOMPLETE IMAGE ANALYSIS (HARRIS)

INTERCORRELATIONS OF PRIMARY FACTORS

		1	2	3	4
FACTOR	1	1.000			
FACTOR	2	0.350	1.000		
FACTOR	3	0.315	0.582	1.000	
FACTOR	4	0.010	-0.088	0.011	1.000

EIGHT PHYSICAL VARIABLES

INPUT IS A CORRELATION MATRIX

IMAGE ANALYSIS FOLLOWED BY ORTHOTRAN SOLUTION

INCOMPLETE IMAGE ANALYSIS (HARRIS)

DIRECT AND JOINT PROPORTIONATE CONTRIBUTIONS OF FACTORS TO TOT. COMMON VARIANCE

	DIRECT	+	JOINT	=	TOTAL
1	(0.587)	+	(0.001)	=	0.589
2	(0.182)	+	(0.001)	=	0.183
3	(0.175)	+	(0.000)	=	0.175
4	(0.082)	+	(-0.029)	=	0.053

ORTHOMAX CRITERION IS = 1.00
WITH A NORMALIZED MATRIX

INCOMPLETE IMAGE ANALYSIS (HARRIS)

FACTOR SCORE WEIGHT MATRIX

		1	2	3	4
VAR1	1	0.342	-0.380	0.213	-0.963
VAR2	2	0.256	0.256	-0.047	1.032
VAR3	3	0.192	-0.003	0.204	0.710
VAR4	4	0.306	0.152	-0.348	-0.847
VAR5	5	0.007	-0.511	-0.347	-0.275
VAR6	6	-0.025	-0.625	0.218	0.047
VAR7	7	-0.017	0.176	-0.688	0.012
VAR8	8	-0.046	-0.011	-0.216	0.556
		1	2	3	4

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

ITERATION 7

PRIOR ESTIMATES OF COMMUNALITIES		PRESENT ESTIMATES OF COMMUNALITIES		DIFFERENCES	
1	0.8381526	1	0.8381205	1	0.000032
2	0.8904025	2	0.8905722	2	-0.000170
3	0.8190014	3	0.8189302	3	0.000071
4	0.8067604	4	0.8067293	4	0.000031
5	0.8795411	5	0.8802146	5	-0.000674
6	0.6393320	6	0.6391971	6	0.000135
7	0.5824411	7	0.5821577	7	0.000283
8	0.4998355	8	0.4998133	8	0.000022

SUM OF DIFFERENCE SQUARES =0.00000059

NUMBER OF ITERATIONS IS 7

RESIDUALS OF FINAL TWO COMMUNALITY ESTIMATES =0.0000890002

PRIOR ROOTS		PRESENT ROOTS		DIFFERENCES	
1	2.436771	1	2.436760	1	0.000012
2	1.436035	2	1.436038	2	-0.000003

SUM OF ROOT DIFFERENCE SQUARES =0.00000000

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

COMMUNALITY ESTIMATES

		1
VAR1	1	0.838
VAR2	2	0.891
VAR3	3	0.819
VAR4	4	0.807
VAR5	5	0.881
VAR6	6	0.639
VAR7	7	0.582
VAR8	8	0.500

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

OBLIQUE PRIMARY PATTERN SOLUTION MATRIX(ORTHOTRAN)

		1	2
VAR1	1	0.877	0.079
VAR2	2	0.944	-0.000
VAR3	3	0.916	-0.025
VAR4	4	0.873	0.052
VAR5	5	-0.000	0.938
VAR6	6	-0.012	0.805
VAR7	7	-0.070	0.793
VAR8	8	0.093	0.659
		1	2

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

VARIABLE COMPLEXITY(APPROXIMATE NUMBER

OF FACTORS THAT A VARIABLE LOADS ON

VAR1	1	1.016
VAR2	2	1.000
VAR3	3	1.002
VAR4	4	1.007
VAR5	5	1.000
VAR6	6	1.000
VAR7	7	1.016
VAR8	8	1.040

AVERAGE VARIABLE COMPLEXITY = 1.010

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

OBLIQUE REFERENCE STRUCTURE SOLUTION

		1	2
VAR1	1	0.778	0.070
VAR2	2	0.838	-0.000
VAR3	3	0.813	-0.023
VAR4	4	0.775	0.046
VAR5	5	-0.000	0.833
VAR6	6	-0.011	0.715
VAR7	7	-0.062	0.704
VAR8	8	0.083	0.585
		1	2

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

INTERCORRELATIONS OF PRIMARY FACTORS

		1	2
FACTOR	1	1.000	
FACTOR	2	0.461	1.000
		1	2

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

DIRECT AND JOINT PROPORTIONATE CONTRIBUTIONS

	DIRECT	+	JOINT	=	TOTAL
1	(0.448)	+	(0.196)	=	0.644
2	(0.356)	+	(-0.000)	=	0.356

ORTHOMAX CRITERION IS = 1.00
WITH A NORMALIZED MATRIX

ALPHA FACTOR ANALYSIS (KAISER-CAFFREY)

FACTOR SCORE WEIGHT MATRIX

		1	2
VAR1	1	-0.243	0.046
VAR2	2	-0.404	-0.157
VAR3	3	-0.192	0.078
VAR4	4	-0.199	-0.012
VAR5	5	-0.017	-0.442
VAR6	6	-0.000	-0.162
VAR7	7	0.009	-0.120
VAR8	8	0.009	-0.115
		1	2