Intelligent Financial Planning Models Using PROC LP

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Introduction:

Financial management requires planning based on the simultaneous consideration of the investment, financing and dividend options facing the firm. The LONGER model (Myers and Pogue 1974) is a linear programming model that fulfills this need.

In this report we introduce the concepts behind financial models based on linear programming. We show how to use PROC LP and PROC COMPUTAB to define an lp model for answering financial questions and how to report the solution.

Examples:

Consider a firm which has to decide how much to invest or borrow in the coming year.

Let x = New investment in millions of dollars
y = New borrowing in millions of dollars.

Assume:

1. Available investment opportunities can absorb $ 1 million at most.
2. The investment opportunity generates a net present value of -$ 0.10 per invested dollar.
3. New debt is limited to 40 percent of new investment.
4. The firm has $ 800,000 in cash.
5. Marginal corporate tax rate is .5.
6. The firm bases decisions on maximizing the value of firm (which includes the present value of its tax shields.

The question of interest is:
What levels of new investment and borrowing maximize the value of the firm?

If the current value of the firm is V then the the values of x and y that maximize

$$V - 0.1 x + 0.5 y$$

and satisfy the constraints imposed by the assumptions maximize the value of the firm.

This can be solve with PROC LP.

The Linear Programming Formulation is:

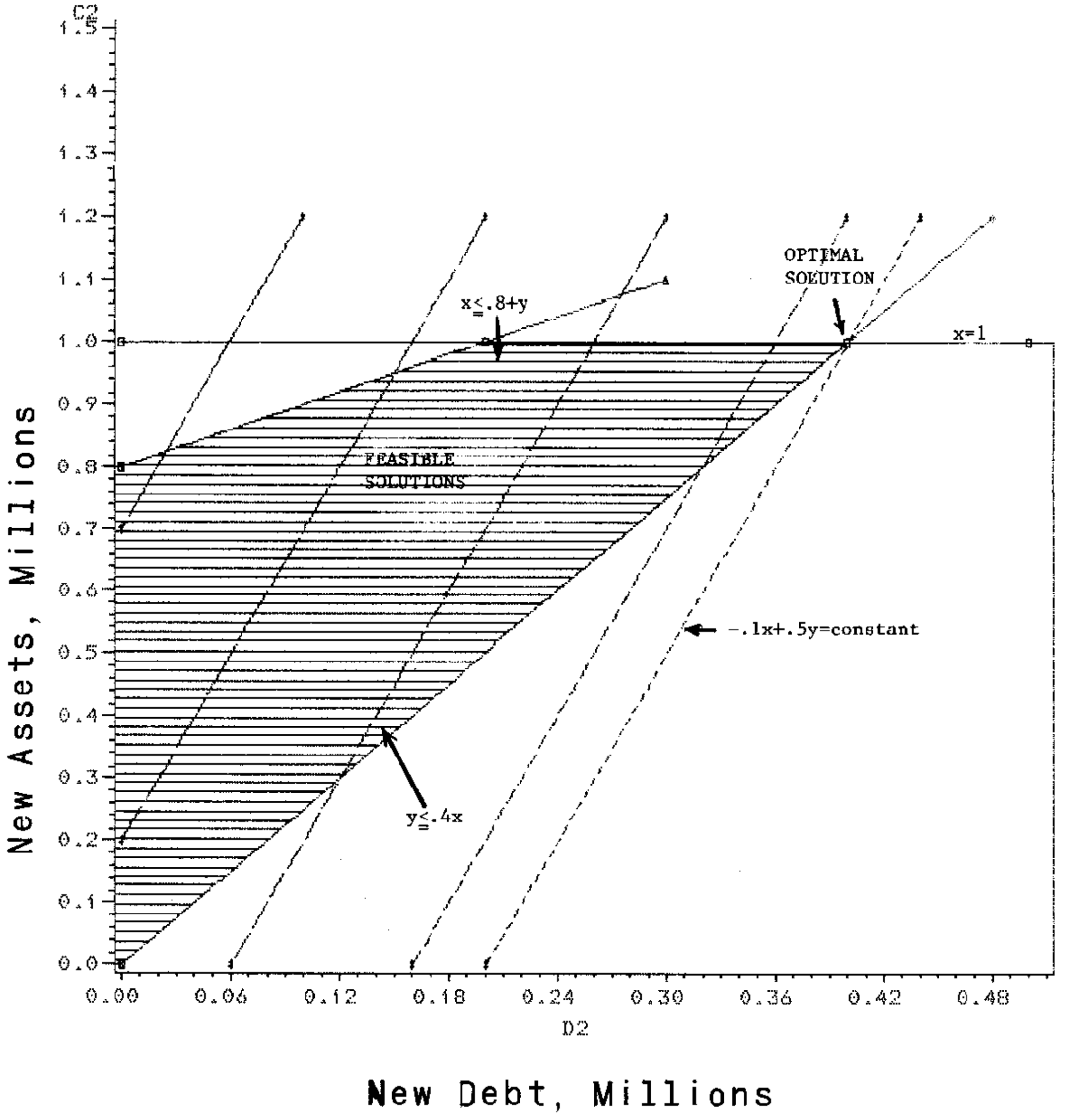
Maximize $-0.1 x + 0.5 y$

Subject to the constraints:

```
assumption 1:      x  <= 1
assumption 3:      y  <= 0.4 x
assumption 4:      x  <= y + .8
```

These constraints are geometrically represented as:

GEOMETRIC REPRESENTATION OF PROBLEM



To solve this problem with PROC LP you save the model in a SAS data set then call the LP procedure.

```
DATA; INPUT ID $ X Y TYPE $ RHS;
CARDS;
VALUE  -.1  .5  MAX   .
ASSUM1   1   0   <=   1
ASSUM3 -.4   1   <=   0
ASSUM4   1  -1   <=  .8

PROC LP;
   VAR X Y;
   TYPE TYPE;
   RHS RHS;
   ID ID;
```

This program produces the following:

```
L I N E A R   P R O G R A M M I N G   P R O C E D U R E
                 PROBLEM  SUMMARY

MAX VALUE              OBJECTIVE FUNCTION
RHS                          RHS  VARIABLE
TYPE                        TYPE  VARIABLE
PROBLEM DENSITY                      0.633

VARIABLE TYPE                       NUMBER

STRUCTURAL
 NONNEGATIVE                             2

LOGICAL
 SLACK                                   3

TOTAL                                    5

CONSTRAINT TYPE                     NUMBER

LE                                       3
FREE                                     1

TOTAL                                    4

                SOLUTION  SUMMARY

             TERMINATED SUCCESSFULLY

OBJECTIVE VALUE                   0.100000

PHASE 1 ITERATIONS                       0
PHASE 2 ITERATIONS                       3
INITIAL B.F. VARIABLES                   3
TIME USED (SECS)                      0.03
NUMBER OF INVERSIONS                     1
```

VARIABLE SUMMARY

COL.	VARIABLE NAME	STATUS	TYPE	PRICE	ACTIVITY	REDUCED COST
1	X	BASIC	NON-NEG	-0.1	1.000000	0
2	Y	BASIC	NON-NEG	0.5	0.400000	0
3	ROW1		SLACK	0	0	-0.100000
4	ROW2		SLACK	0	0	-0.500000
5	ROW3	BASIC	SLACK	0	0.200000	0

CONSTRAINT SUMMARY

CONSTRAINT ROW	ID	TYPE	S/S COL	RHS	ACTIVITY	DUAL ACTIVITY
1	ASSUM1	LE	3	1.000000	1.000000	0.100000
2	ASSUM3	LE	4	0	0	0.500000
3	ASSUM4	LE	5	0.800000	0.600000	0
4	VALUE	OBJECTIVE		0.100000	0.100000	0

Since the variable x, investment, has an activity of 1 and the variable y, borrowing, has an activity of .4 the firm can maximize the its value in this scenario by investing $ 1 million of which $ .4 million is borrowed.

The objective activity of .1 shows that following this level of investment and borrowing increases the value of the firm by $ .1 million.

The dual activity (shadow price) associated with the constraint on the size of the available investment (ASSUM1) tells you the marginal contribution of that investment to the present value of the firm. Since it accounts for the firm's financing decisions it is the adjusted present value of the investment opportunity. In this case it is $ .1 million.

This Model can be extended to include other aspects of corporate finance:

1. dividend policy
2. debt policy
3. liquidity
4. project interactions

For example consider the following pro-forma financial statements for 1983 for the Executive Fruit Company. This is merely an accounting model.

EXECUTIVE FRUIT COMPANY
(ALL FIGURES IN THOUSANDS OF DOLLARS)

	1983
INCOME STATEMENT	
REVENUE (REV)	2808.0
COST OF GOODS SOLD (CGS)	2527.0
EARNINGS BEFORE INTEREST AND TAXES (ERBINTX)	281.0
INTEREST (INT)	59.0
EARNINGS BEFORE TAX (EBT)	222.0
TAX (TAX)	111.0
NET INCOME (NET)	111.0
SOURCES AND USES OF FUNDS	
SOURCES	
NET INCOME (NET)	111.0
DEPRECIATION (DEP)	104.0
OPERATING CASH FLOW (OPCSFL)	215.0
BORROWING (DD)	255.6
STOCK ISSUES (SI)	0.0
TOTAL SOURCES (TSOURCE)	470.6
USES	
TOTAL INVESTMENTS (INV)	60.0
INCREASE IN NET WORKING CAPITAL (DNWC)	344.0
TOTAL DIVIDENDS (DIV)	66.6
TOTAL USES (TUSES)	470.6
BALANCE SHEET	
ASSETS	
NET WORKING CAPITAL (NWC)	261.1
FIXED ASSETS (FA)	1038.9
TOTAL ASSETS (TASSET)	1300.0
LIABILITIES	
DEBT (DEBT)	655.6
BOOK EQUITY (EQUITY)	644.4
TOTAL LIABILITIES (TOTLIB)	1300.0

This report, taken from Brealey and Myers (1981), is the result of a financial model that incorporates the firm's 1982 position, current market conditions, and management decisions, to project the firm's 1983 position.

The model is summarized with these equations:

```
REV = forecast by the model user
CGS = a(1) REV
INT = a(2) DEBT
TAX = a(3) (REV - CGS - INT)
NET = REV - CGS - INT - TAX               (accounting identity)

DEP = a(4) FA
DD = DNWC + INV + DIV - NET - DEP - SI    (accounting identity)
SI = decision of model user
DNWC = NWC - NWC(-1)                      (accounting identity)
INV = DEP + FA - FA(-1)                   (accounting identity)
DIV = a(5) NET

NWC = a(6) REV
FA = a(7) REV
DEBT = DD + DEBT(-1)                      (accounting identity)
EQUITY = EQUITY(-1) + NET - DIV + SI      (accounting identity)
```

The (-1) indicates the previous years position.

Coefficients a(1) through a(7) correspond to existing market conditions and corporate operating policy.

For example, a(2) = interest rate, and
a(5) = corporate payout rate.

The Executive Fruit Company balance sheets are produced by assuming that REV = 2808 in 1982 and that

```
a(1) = .9     a(2) = .09     a(3) = .5
a(4) = .1     a(5) = .6      a(6) = .093
a(7) = .37.
```

The position of Executive Fruit at the begining of 1983 is summarized by:

```
DEBT(-1)   = 400
FA(-1)     = 800
EQUITY(-1) = 600
NWC(-1)    = 200
```

Now we want to show how to combine this predictive model with the linear programming model. First, we present a scenario that poses a question regarding the distribution of investment funds. Then, we use PROC LP to identify the optimal distribution of investment funds. In the last step we show how PROC COMPUTAB can be used to present the solution in a balance sheet format.

Consider the following scenario:

(1) Executive Fruit has an opportunity to invest in two investments, A and B, which would increase revenues by:

	1984	1985
Investment A	.14	.70
Investment B	.10	.74

in dollars, for each dollar of investment in 1983.

(2) Investment A can absorb at most $ .8 million and investment B can absorb at most $ 1 million.

(3) Company policy is to limit new debt to .4 of total investments.

(4) Executive Fruit expects revenues to grow by .1 regardless of investment in either investment A or investment B.

(5) The current discount rate for investments of equal risk to investment A and investment B is .9.

The question of interest is how much should be invested in 1983 so as to maximize the value of the firm?

To answer this we incorporate assumptions (1)-(5) associated with the two investments into the single period model above.

We use the variable names identified on the Executive Fruit balance sheet with a 1, 2 or 3 appended to them. This number serves to distinguish period.

The DATA STEP that follows saves the model in the SAS data set named CONST.

```
DATA CONST.
  INPUT ID $ RHS TYPE $
        REV1= COS1= ERBINTX1= TAX1= NET1= DEP1= OPCSFL1= DD1=
                    SI1= TSOURCE1= DNWC1= INV1= DIV1=
                    TUSES1= NWC1= FA1= TASSETS1= DEBT1= INT1=
                    EQUITY1= TOTLIB1=
        REV2= COS2= ERBINTX2= TAX2= NET2= DEP2= OPCSFL2= DD2=
                    SI2= TSOURCE2= DNWC2= INV2= DIV2=
                    TUSES2= NWC2= FA2= TASSETS2= DEBT2= INT2=
                    EQUITY2= TOTLIB2=
        REV3= COS3= ERBINTX3= TAX3= NET3= DEP3= OPCSFL3= DD3=
                    SI3= TSOURCE3= DNWC3= INV3= DIV3=
                    TUSES3= NWC3= FA3= TASSETS3= DEBT3= INT3=
                    EQUITY3= TOTLIB3=
        INVESTA= INVESTB=
        FA0= NWC0= EQUITY0=
        DEBT0=

CARDS;


REV1      2808 EQ REV1=1
CGS1      0    EQ CGS1=1 REV1=-.9
ERBINTX1  0    EQ ERBINTX1=1 REV1=-1 CGS1=1
INT1      0    EQ INT1=1 DEBT1=-0.09
TAX1      0    EQ TAX1=1 REV1=-0.5 CGS1=0.5 INT1=0.5
NET1      0    EQ NET1=1 REV1=-1   CGS1=1   INT1=1    TAX1=1
DEP1      0    EQ DEP1=1 FA1=-0.1
DD1       0    EQ DD1=1  DNWC1=-1 INV1=-1 DIV1=-1 NET1=1 DEP1=1 SI1=1
DNWC1     0    EQ DNWC1=1 NWC1=-1 NWC0=1
INV1      0    EQ INV1=1  DEP1=-1 FA1=-1 FA0=1
DIV1      0    EQ DIV1=1 NET1=-.6
NWC1      0    EQ NWC1=1 REV1=-0.093 SI1=-1
FA1       0    EQ FA1=1  REV1=-0.37 INVESTA=-1 INVESTB=-1
DEBT1     0    EQ DEBT1=1 DD1=-1 DEBT0=-1
EQUITY1   0    EQ EQUITY1=1 NET1=-1 DIV1=1 SI1=-1 EQUITY0=-1
OPCSFL1   0    EQ OPCSFL1=1 NET1=-1 DEP1=-1
TSOURCE1  0    EQ TSOURCE1=1 OPCSFL1=-1 SI1=-1 DD1=-1
TUSES1    0    EQ TUSES1=1 DNWC1=-1 INV1=-1 DIV1=-1
TASSETS1  0    EQ TASSETS1=1 NWC1=-1 FA1=-1
TOTLIB1   0    EQ TOTLIB1=1 DEBT1=-1 EQUITY1=-1


REV2      0    EQ REV2=1 REV1=-1.1 INVESTA=-.14 INVESTB=-.1
CGS2      0    EQ CGS2=1 REV2=-.9
ERBINTX2  0    EQ ERBINTX2=1 REV2=-1 CGS2=1
INT2      0    EQ INT2=1 DEBT2=-0.09
TAX2      0    EQ TAX2=1 REV2=-0.5 CGS2=0.5 INT2=0.5
NET2      0    EQ NET2=1 REV2=-1   CGS2=1   INT2=1    TAX2=1
DEP2      0    EQ DEP2=1 FA2=-0.1
DD2       0    EQ DD2=1  DNWC2=-1 INV2=-1 DIV2=-1 NET2=1 DEP2=1 SI2=1
DNWC2     0    EQ DNWC2=1 NWC2=-1 NWC1=1
INV2      0    EQ INV2=1  DEP2=-1 FA2=-1 FA1=1
DIV2      0    EQ DIV2=1 NET2=-.6
NWC2      0    EQ NWC2=1 REV2=-0.093 SI2=-1
FA2       0    EQ FA2=1  REV2=-0.37
DEBT2     0    EQ DEBT2=1 DD2=-1 DEBT1=-1
EQUITY2   0    EQ EQUITY2=1 NET2=-1 DIV2=1 SI2=-1 EQUITY1=-1
OPCSFL2   0    EQ OPCSFL2=1 NET2=-1 DEP2=-1
TSOURCE2  0    EQ TSOURCE2=1 OPCSFL2=-1 SI2=-1 DD2=-1
TUSES2    0    EQ TUSES2=1 DNWC2=-1 INV2=-1 DIV2=-1
TASSETS2  0    EQ TASSETS2=1 NWC2=-1 FA2=-1
TOTLIB2   0    EQ TOTLIB2=1 DEBT2=-1 EQUITY2=-1


REV3      0    EQ REV3=1 REV2=-1.1 INVESTA=-.7 INVESTB=-.74
CGS3      0    EQ CGS3=1 REV2=-.9
ERBINTX3  0    EQ ERBINTX3=1 REV3=-1 CGS3=1
INT3      0    EQ INT3=1 DEBT3=-0.09
TAX3      0    EQ TAX3=1 REV3=-0.5 CGS3=0.5 INT3=0.5
NET3      0    EQ NET3=1 REV3=-1   CGS3=1   INT3=1    TAX3=1
DEP3      0    EQ DEP3=1 FA3=-0.1
DD3       0    EQ DD3=1  DNWC3=-1 INV3=-1 DIV3=-1 NET3=1 DEP3=1 SI3=1
DNWC3     0    EQ DNWC3=1 NWC3=-1 NWC2=1
INV3      0    EQ INV3=1  DEP3=-1 FA3=-1 FA2=1
DIV3      0    EQ DIV3=1 NET3=-.6
NWC3      0    EQ NWC3=1 REV3=-0.093 SI3=-1
FA3       0    EQ FA3=1  REV3=-0.37
DEBT3     0    EQ DEBT3=1 DD3=-1 DEBT2=-1
EQUITY3   0    EQ EQUITY3=1 EQUITY2=-1 NET3=-1 DIV3=1 SI3=-1
OPCSFL3   0    EQ OPCSFL3=1 NET3=-1 DEP3=-1
TSOURCE3  0    EQ TSOURCE3=1 OPCSFL3=-1 SI3=-1 DD3=-1
TUSES3    0    EQ TUSES3=1 DNWC3=-1 INV3=-1 DIV3=-1
TASSETS3  0    EQ TASSETS3=1 NWC3=-1 FA3=-1
TOTLIB3   0    EQ TOTLIB3=1 DEBT3=-1 EQUITY3=-1
INVALIM   97   LE INVESTA=1
INVBLIM   100  LE INVESTB=1
NEWDEBT   0    LE INV1=-.4 INVESTA=1 INVESTB=1 NWC=-1
DEBT0     400  EQ DEBT0=1
FA0       800  EQ FA0=1
EQUITY0   600  EQ EQUITY0=1
NWC0      200  EQ NWC0=1
SI1       0    EQ SI1=1
SI2       0    EQ SI2=1
SI3       0    EQ SI3=1
```

To help you interpret the model consider the observation where the variable ID equals REV2 namely.

```
REV2   0   EQ REV2=1 REV1=-1.1 INVESTA=-.14 INVESTB=-.1
```

This constraint defines the revenue in the second period. The constraint is interpreted as the equation:

$$0 = REV2 - 1.1\ REV1 - .14\ INVESTA - .1\ INVESTB,$$

or rewriting it as

$$REV2 = 1.1\ REV1 + .14\ INVESTA + .1\ INVESTB.$$

As the assumptions state, the revenue in the second period is .1 greater than the revenue in the first period plus .14 of investment A and .1 of investment B.

The objective of maximizing the present value of the firm is represented in the SAS data set generated by the code:

```
DATA OBJ; INPUT ID $ RHS TYPE $
              TASSETS1 TASSETS2 TASSETS3
              DD1 DD2 DD3
              DIV1 DIV2 DIV3
              SI1 SI2 SI3
              RET1 RET2 RET3
              INVESTA INVESTB;
CARDS,
RET1      0 EQ  1 0 0 .5 0   0 0 0 0   0    0     0 -1  0  0  0  0
RET2      0 EQ  0 1 0  0 .5  0 0 0 0   0    0     0  0 -1  0  0  0
RET3      0 EQ  0 0 1  0 0  .5 0 0 0   0    0     0  0  0 -1  0  0
OBJECTIV    MAX 0 0 0  0 0   0 0 0 0 -.1  -.1   -.1  1 .9 .81 -1 -1
```

PROC LP can be used to maximize this objective within the financial constraints imposed by the operating characteristics of the firm.

First concatenate the constraint and objective data sets, then call the LP procedure.

```
DATA; SET CONST OBJ;

PROC LP POUT=PRIMAL;
   TYPE TYPE;
   RHS RHS;
   ID ID.
```

This produces the output:

```
                L I N E A R   P R O G R A M M I N G   P R O C E D U R E

                               PROBLEM  SUMMARY

              MAX OBJECTIV          OBJECTIVE FUNCTION
              RHS                          RHS  VARIABLE
              TYPE                         TYPE VARIABLE
              PROBLEM DENSITY                     0 041

              VARIABLE TYPE                      NUMBER

              STRUCTURAL
               NONNEGATIVE                           72

              LOGICAL
               SLACK                                  3

              TOTAL                                  75

              CONSTRAINT TYPE                    NUMBER

              LE                                      3
              EQ                                     70
              FREE                                    1

              TOTAL                                  74


                L I N E A R   P R O G R A M M I N G   P R O C E D U R E

                               SOLUTION  SUMMARY

                           TERMINATED SUCCESSFULLY

              OBJECTIVE VALUE                4089.767

              PHASE 1 ITERATIONS                   68
              PHASE 2 ITERATIONS                    3
              INITIAL B.F. VARIABLES               18
              TIME USED (SECS)                   5.76
              COMPRESSIONS/ITERATION             0.06

                               VARIABLE  SUMMARY

      VARIABLE                                                        REDUCED
  COL NAME      STATUS   TYPE          PRICE       ACTIVITY             COST

    1 REV1      BASIC   NON-NEG            0       2808.000                0
    2 CGS1      BASIC   NON-NEG            0       2527.200                0
    3 ERBINTX1  BASIC   NON-NEG            0        280.800                0
    4 TAX1      BASIC   NON-NEG            0        105.908                0
    5 NET1      BASIC   NON-NEG            0        105.908                0
    6 DEP1      BASIC   NON-NEG            0        112.599                0
    7 OPCSFL1   BASIC   NON-NEG            0        218.507                0
    8 DD1       BASIC   NON-NEG            0        344.979                0
    9 SI1       DEGEN   NON-NEG         -0.1              0                0
   10 TSOURCE1  BASIC   NON-NEG            0        563.675                0
   11 DNWC1     BASIC   NON-NEG            0      61.144000                0
   12 INV1      BASIC   NON-NEG            0        438.589                0
   13 DIV1      BASIC   NON-NEG            0      64.141992                0
   14 TUSES1    BASIC   NON-NEG            0        563.675                0
   15 NWC1      BASIC   NON-NEG            0        261.144                0
   16 FA1       BASIC   NON-NEG            0       1125.990                0
   17 TASSETS1  BASIC   NON-NEG            0       1387.134                0
   18 DEBT1     BASIC   NON-NEG            0        744.373                0
   19 INT1      BASIC   NON-NEG            0      66.993561                0
   20 EQUITY1   BASIC   NON-NEG            0        642.761                0
   21 TOTLIB1   BASIC   NON-NEG            0       1387.134                0
   22 REV2      BASIC   NON-NEG            0       3100.988                0
   23 CGS2      BASIC   NON-NEG            0       2790.885                0
   24 ERBINTX2  BASIC   NON-NEG            0        310.099                0
   25 TAX2      BASIC   NON-NEG            0        121.652                0
   26 NET2      BASIC   NON-NEG            0        121.652                0
   27 DEP2      BASIC   NON-NEG            0        114.796                0
   28 OPCSFL2   BASIC   NON-NEG            0        236.299                0
   29 DD2               NON-NEG            0              0        -0.545670
   30 SI2       DEGEN   NON-NEG         -0.1              0                0
   31 TSOURCE2  BASIC   NON-NEG            0        236.299                0
   32 DNWC2     BASIC   NON-NEG            0      27.267421                0
   33 INV2      BASIC   NON-NEG            0        136.110                0
```

L I N E A R   P R O G R A M M I N G   P R O C E D U R E

VARIABLE SUMMARY

COL	VARIABLE NAME	STATUS	TYPE	PRICE	ACTIVITY	REDUCED COST
34	DIV2	BASIC	NON-NEG	0	72.981422	0
35	TUSES2	BASIC	NON-NEG	0	236.289	0
36	NWC2	BASIC	NON-NEG	0	286.391	0
37	FA2	BASIC	NON-NEG	0	1147.364	0
38	TASSETS2	BASIC	NON-NEG	0	1435.755	0
39	DEBT2	BASIC	NON-NEG	0	744.373	0
40	INT2	BASIC	NON-NEG	0	66.993561	0
41	EQUITY2	BASIC	NON-NEG	0	691.382	0
42	TOTLIB2	BASIC	NON-NEG	0	1435.755	0
43	REV3	BASIC	NON-NEG	0	3472.004	0
44	CGS3	BASIC	NON-NEG	0	2790.885	0
45	ERBINTX3	BASIC	NON-NEG	0	681.119	0
46	TAX3	BASIC	NON-NEG	0	304.919	0
47	NET3	BASIC	NON-NEG	0	304.919	0
48	DEP3	BASIC	NON-NEG	0	128.464	0
49	OPCSFL3	BASIC	NON-NEG	0	482.288	0
50	DD3	BASIC	NON-NEG	0	49.854868	0
51	SI3	DEGEN	NON-NEG	-0.1	0	0
52	TSOURCE3	BASIC	NON-NEG	0	488.159	0
53	DNWC3	BASIC	NON-NEG	0	34.504920	0
54	INV3	BASIC	NON-NEG	0	265.742	0
55	DIV3	BASIC	NON-NEG	0	182.892	0
56	TUSES3	BASIC	NON-NEG	0	488.159	0
57	NWC3	BASIC	NON-NEG	0	322.895	0
58	FA3	BASIC	NON-NEG	0	1284.641	0
59	TASSETS3	BASIC	NON-NEG	0	1607.538	0
60	DEBT3	BASIC	NON-NEG	0	794.228	0
61	INT3	BASIC	NON-NEG	0	71.490489	0
62	EQUITY3	BASIC	NON-NEG	0	813.310	0
63	TOTLIB3	BASIC	NON-NEG	0	1607.538	0
64	INVESTA	BASIC	NON-NEG	-1	87.000000	0
65	INVESTB	BASIC	NON-NEG	-1	0.030190	0
66	FA0	BASIC	NON-NEG	0	800.000	0

L I N E A R   P R O G R A M M I N G   P R O C E D U R E

VARIABLE SUMMARY

COL	VARIABLE NAME	STATUS	TYPE	PRICE	ACTIVITY	REDUCED COST
67	NWC0	BASIC	NON-NEG	0	200.000	0
68	EQUITY0	BASIC	NON-NEG	0	600.000	0
69	DEBT0	BASIC	NON-NEG	0	400.000	0
70	RET1	BASIC	NON-NEG	1	1559.321	0
71	RET2	BASIC	NON-NEG	0.9	1435.755	0
72	RET3	BASIC	NON-NEG	0.81	1682.455	0
73	ROW61		SLACK	0	0	-0.082088
74	ROW62	BASIC	SLACK	0	99.969810	0
75	ROW63	BASIC	SLACK	0	99.405404	0

L I N E A R   P R O G R A M M I N G   P R O C E D U R E

CONSTRAINT SUMMARY

CONSTRAINT ROW	ID	TYPE	S/S COL	RHS	ACTIVITY	DUAL ACTIVITY
1	REV1	EQ		2808.000	2808.000	1.640047
2	CGS1	EQ		0	0	0.107090
3	ERBINTX1	EQ		0	0	0
4	INT1	EQ		0	0	0.107090
5	TAX1	EQ		0	0	0.214181
6	NET1	EQ		0	0	-0.214181
7	DEP1	EQ		0	0	0
8	DD1	EQ		0	0	0.535452
9	DNWC1	EQ		0	0	0.535452
10	INV1	EQ		0	0	0.535452
11	DIV1	EQ		0	0	0.535452
12	NWC1	EQ		0	0	0.513758
13	FA1	EQ		0	0	0.513758
14	DEBT1	EQ		0	0	0.035452
15	EQUITY1	EQ		0	0	0
16	OPCSFL1	EQ		0	0	0
17	TSOURCE1	EQ		0	0	0
18	TUSES1	EQ		0	0	0
19	TASSETS1	EQ		0	0	1.000000
20	TOTLIB1	EQ		0	0	0
21	REV2	EQ		0	0	1.284497
22	CGS2	EQ		0	0	0.204337
23	ERBINTX2	EQ		0	0	0
24	INT2	EQ		0	0	0.204337
25	TAX2	EQ		0	0	0.408674
26	NET2	EQ		0	0	-0.408674
27	DEP2	EQ		0	0	0
28	DD2	EQ		0	0	1.021684
29	DNWC2	EQ		0	0	1.021684
30	INV2	EQ		0	0	1.021684
31	DIV2	EQ		0	0	1.021684
32	NWC2	EQ		0	0	1.509250
33	FA2	EQ		0	0	1.509250

L I N E A R   P R O G R A M M I N G   P R O C E D U R E

CONSTRAINT SUMMARY

CONSTRAINT ROW	ID	TYPE	S/S COL	RHS	ACTIVITY	DUAL ACTIVITY
34	DEBT2	EQ		0	0	0.025814
35	EQUITY2	EQ		0	0	0
36	OPCSFL2	EQ		0	0	0
37	TSOURCE2	EQ		0	0	0
38	TUSES2	EQ		0	0	0
39	TASSETS2	EQ		0	0	0.900000
40	TOTLIB2	EQ		0	0	0
41	REV3	EQ		0	0	0.483497
42	CGS3	EQ		0	0	0.082495
43	ERBINTX3	EQ		0	0	0
44	INT3	EQ		0	0	0.082495
45	TAX3	EQ		0	0	0.164969
46	NET3	EQ		0	0	-0.164969
47	DEP3	EQ		0	0	0
48	DD3	EQ		0	0	0.412424
49	DNWC3	EQ		0	0	0.412424
50	INV3	EQ		0	0	0.412424
51	DIV3	EQ		0	0	0.412424
52	NWC3	EQ		0	0	1.222424
53	FA3	EQ		0	0	1.222424
54	DEBT3	EQ		0	0	0.007425525
55	EQUITY3	EQ		0	0	0
56	OPCSFL3	EQ		0	0	0
57	TSOURCE3	EQ		0	0	0
58	TUSES3	EQ		0	0	0
59	TASSETS3	EQ		0	0	0.810000
60	TOTLIB3	EQ		0	0	0
61	INVALIM	LE	73	87.000000	87.000000	0.082088
62	INVBLIM	LE	74	100.000000	0.030190	0
63	NEWDEBT	LE	75	0	-99.405404	0
64	DEBT0	EQ		400.000	400.000	0.035452
65	FA0	EQ		800.000	800.000	-0.535452
66	EQUITY0	EQ		600.000	600.000	0
67	NWC0	EQ		200.000	200.000	-0.535452

L I N E A R   P R O G R A M M I N G   P R O C E D U R E

CONSTRAINT SUMMARY

CONSTRAINT ROW	ID	TYPE	S/S COL	RHS	ACTIVITY	DUAL ACTIVITY
68	SI1	EQ		0	0	-0.121694
69	SI2	EQ		0	0	0.887576
70	SI3	EQ		0	0	0.710000
71	RET1	EQ		0	0	-1.000000
72	RET2	EQ		0	0	-0.900000
73	RET3	EQ		0	0	-0.810000
74	OBJECTIV	OBJECTIVE		4085.767	4085.767	0

Notice that the value of the objective, namely the maximization of the firm's present value is $4.086 million. To achieve this objective we must invest $ 87,000 in investment A and a token $ 30 in investment B. The solution tells you to invest as much as possible in A. The limit on investment A, specified in constraint row 61, can be seen to be at its limit in the optimal solution.

The dual activity for this constraint gives the marginal value of increasing investment in A beyond its optimal level. Since it takes into account all the financing decisions that have been included in the model it is the adjusted present value of that investment.

Lastly, we use PROC COMPUTAB to report the solution. The primal output from PROC LP is in a SAS data set containing the values of the variables at optimality. Each observation contains the value of a single variable.

The desired format of the Sources and Uses Table is a row by column arrangement with model variables composing the rows and the periods under study composing the columns. PROC COMPUTAB automatically transposes its input data set; therefore, the primal output data set must be re-shaped before invoking COMPUTAB.

Create a new variable, PERIOD, that will identify each observation with the period to which it belongs. TRANSPOSE the primal output data set BY PERIOD to produce an input data set for PROC COMPUTAB. The input data set contains one observation for each period. Each observation contains the values of the model variables for the period.

The program that follows does this:

```
DATA ; SET PRIMAL;
   LENGTH PERIOD $ 8;
   IF   _N_ <22      THEN PERIOD='PERIOD1';
   ELSE IF _N_ <43 THEN PERIOD='PERIOD2';
   ELSE IF _N_ <64 THEN PERIOD='PERIOD3';
   ELSE                 PERIOD='PERIOD1';
   _VAR_ =COMPRESS(_VAR_,'123');

PROC SORT; BY PERIOD;

PROC TRANSPOSE OUT=COMPIN;
   ID _VAR_;
   VAR _VALUE_;
   BY PERIOD;
```

When you call PROC COMPUTAB include the period names in COLUMN statements and the model variable (without appended period numbers) in ROW statements.

The period numbers were removed by the COMPRESS function and PROC TRANSPOSE used the resulting names as variable names on the COMPUTAB input data set.

Calculate new rows and columns using SAS program statements in the input block, row blocks, or column blocks. Here we calculate earnings before tax (EBT) in the input block and other investment capacity (INVO) in a row block. The final product is a report that displays the optimal distribution of investments and the effect of these investments on the income statement, sources and uses of funds, and balance sheet of the firm.

The call of PROC COMPUTAB is:

```
OPTIONS NONOTES;
PROC COMPUTAB DATA=COMPIN;
   TITLE1 EXECUTIVE FRUIT COMPANY;
   TITLE2 PRO FORMA STATEMENTS FOR THE;
   TITLE3 CORPORATE 3-YEAR STRATEGIC PLAN;
   TITLE4 '(ALL FIGURES IN THOUSANDS OF DOLLARS)';
   COL  PERIOD1 PERIOD2 PERIOD3    / _NAME_ ' ' F=7.1;
   COL  PERIOD1 / '1983';
   COL  PERIOD2 / '1984';
   COL  PERIOD3 / '1985';
   ROW  GROUP1  / 'INCOME STATEMENT' OVP ZERO=' ';
   ROW  REV     / ' REVENUE';
   ROW  CGS     / ' COST OF GOODS SOLD' UL;
   ROW  ERBINTX / ' EARNINGS BEFORE INTEREST AND TAXES';
   ROW  INT     / ' INTEREST' UL;
   ROW  EBT     / ' EARNINGS BEFORE TAX';
   ROW  TAX     / ' TAX' UL;
   ROW  NET     / ' NET INCOME' SKIP;
   ROW  GROUP2  / 'SOURCES AND USES OF FUNDS' OVP ZERO=' ';
   ROW  NET2    / ' SOURCES' '   NET INCOME';
   ROW  DEP     / '   DEPRECIATION' UL;
   ROW  OPCSFL  / '   OPERATING CASH FLOW';
   ROW  DD      / '   BORROWING';
   ROW  SI      / '   STOCK ISSUES' UL;
   ROW  TSOURCE / '   TOTAL SOURCES' DUL SKIP;
   ROW  INVESTA / ' USES' '   INVESTMENT A';
   ROW  INVESTB / '   INVESTMENT B';
   ROW  INVO    / '   OTHER INVESTMENTS' UL;
   ROW  INV     / '   TOTAL INVESTMENTS' SKIP;
   ROW  DNWC    / '   INCREASE IN NET WORKING CAPITAL' SKIP;
   ROW  DIV     / '   TOTAL DIVIDENDS' UL;
   ROW  TUSES   / '   TOTAL USES' DUL SKIP;
   ROW  GROUP3  / 'BALANCE SHEET' OVP ZERO=' ';
   ROW  NWC     / ' ASSETS' '   NET WORKING CAPITAL';
   ROW  FA      / '   FIXED ASSETS' UL;
   ROW  TASSETS / '   TOTAL ASSETS'  SKIP;
   ROW  DEBT    / ' LIABILITIES' '   DEBT';
   ROW  EQUITY  / '   BOOK EQUITY' UL;
   ROW  TOTLIB  / '   TOTAL LIABILITIES' ;

    COL = _N_;
   NET2=NET;
   EBT=ERBINTX-INT;
   ROW : INVO=INV-INVESTA-INVESTB;
```

This program produces the report:

EXECUTIVE FRUIT COMPANY
PRO FORMA STATEMENTS FOR THE
CORPORATE 3-YEAR STRATEGIC PLAN
(ALL FIGURES IN THOUSANDS OF DOLLARS)

	PERIOD1	PERIOD2	PERIOD3
	1983	1984	1985
INCOME STATEMENT			
REVENUE	2908.0	3101.0	3472.0
COST OF GOODS SOLD	2627.2	2790.9	2790.9
EARNINGS BEFORE INTEREST AND TAXES	280.8	310.1	681.1
INTEREST	67.0	67.0	71.5
EARNINGS BEFORE TAX	213.8	243.1	609.6
TAX	106.9	121.6	304.8
NET INCOME	106.9	121.6	304.8
SOURCES AND USES OF FUNDS			
SOURCES			
NET INCOME	106.9	121.6	304.8
DEPRECIATION	112.6	114.7	128.5
OPERATING CASH FLOW	219.5	236.3	433.3
BORROWING	344.4	0.0	49.9
STOCK ISSUES	0.0	0.0	0.0
TOTAL SOURCES	563.9	236.3	483.1
USES			
INVESTMENT A	87.0	0.0	0.0
INVESTMENT B	0.0	0.0	0.0
OTHER INVESTMENTS	351.6	136.1	295.7
TOTAL INVESTMENTS	438.6	136.1	295.7
INCREASE IN NET WORKING CAPITAL	61.1	27.2	84.5
TOTAL DIVIDENDS	64.1	72.9	102.9
TOTAL USES	563.9	236.3	483.1
BALANCE SHEET			
ASSETS			
NET WORKING CAPITAL	261.1	288.4	322.9
FIXED ASSETS	1126.0	1147.4	1284.6
TOTAL ASSETS	1387.1	1435.9	1607.5
LIABILITIES			
DEBT	744.4	744.4	794.2
BOOK EQUITY	642.6	691.4	813.3
TOTAL LIABILITIES	1387.1	1435.9	1607.5

Summary:

We have shown with two examples how linear programming can help the financial planner. The examples indicate how procedures LP and COMPUTAB can be used together to implement the LONGER model proposed by Brealey and Myers.

Furthermore, Brealey and Myers claim that "you need adjusted present value (APV) because it is the only generally reliable approach to capital budgeting when investment decisions have important financing side effects". The LONGER model has the advantage over ad-hoc simulations in that it not only optimizes the value of the firm but also provides APVs for the capital budgeting of investment as exhibited as the shadow prices (dual activities) of the investment constraints.

This is only a start. No model automates entirely the decisions required by the financial planner, or captures all of the issues. The linear programming model we have shown does however give an efficient, intellegent tool for financial planning.

References:

Brealey R., and Myers S. (1981). Principles of Corporate Finance. McGraw-Hill Inc., New York.

Meyers S. and G. Pogue. (1974). "Programming approach to corporate financial management". Journal of Finance, 29: 579 - 599.