A TIME SERIES APPROACH TO MODELING DAILY PEAK ELECTRICITY DEMANDS

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ABSTRACT
Statistical models of peak electricity demand provide the basis for short-term load forecasting and weather normalization applications. Due to the weather sensitive nature of peak demand, a modeling methodology is developed that explicitly accounts for the relationship between peak demand and temperature. By focusing on daily data, this relationship can be identified on a seasonal basis for any given year.

A multi-regression framework is initially formulated that provides the basic structure for a time series modeling approach. The SAS/ETS® ARIMA procedure is used to identify and estimate a Box-Jenkins model that accounts for the non-linear relationship between daily peak demand and temperature through a specification of piecewise-linear transfer functions. Also, since electricity usage varies by day types, pulse intervention variables are introduced to capture the effects of weekends and holidays.

As a final diagnostic check for model adequacy, the estimated residuals from the Box-Jenkins model are subjected to the frequency domain based tests of white noise available in PROC SPECTRA.

INTRODUCTION
Quantitative methods to statistically model and forecast the behavior of a variable over time are generally classified as being either causal or time series approaches. Regression analysis represents a causal, or explanatory, technique that attempts to explain an output variable as a function of one or more input variables. In contrast, time series methods focus entirely on identifying historical patterns of the output variable and require no additional input data. Actually, the time series approach can be further broken down into univariate and multivariate methods. Univariate time series techniques are based solely on the historical time series of a single output variable. Multivariate time series analysis represents a mixture of univariate and causal methods to the extent that it modifies the basic univariate approach to allow for one or more input variables.

The focus of this paper is on a comparison of alternative modeling methodologies as applied toward the development of a statistical model for daily peak megawatt (MW) demands as observed for Florida Power Corporation (FPC). Initially, a regression based model is specified to deal with some of the major issues in modeling daily peaks. As a result of some theoretical and practical problems with this purely causal approach, the basic regression specification is allowed to evolve into the realm of multivariate time series procedures. As part of this model evolution, several alternative models are estimated and tested including a basic multi-regression model using PROC REG, a regression model that adjusts for an autoregressive error structure using PROC AUTOREG, and a Box-Jenkins Autoregressive Integrated Moving-Average (ARIMA) transfer function model using PROC ARIMA.

The estimated models are evaluated based on goodness of fit criteria and tests of white noise on the residual series using PROC SPECTRA.

DATA
Models of peak demand are especially useful for weather normalization and forecasting purposes. Since changing weather conditions represent the primary source of variation in peak demand, the question is often asked, "what would have been the peak demand had normal weather prevailed?" In order to answer this question, statistical models that relate peak demand to one or more weather variables can be developed and used for estimating historical peak demands under a set of alternative weather conditions, rather than those which had actually occurred. By focusing on daily peaks, the models can provide short-term forecasts that will assist in the economic planning and dispatching of electric energy. These same models also yield information concerning trends in the weather responsiveness of peak demands over time which can be incorporated into the long-term forecasting process. Thus, depending on the application, the modeling objectives and approach may vary considerably. Since weather normalization is mainly an exercise in backcasting, or adjusting historical data rather than forecasting, the objective should be to obtain an accurate estimate of the relationship between peak demand and weather, as well as provide a good fit to the historical data. For forecasting purposes the requirements are similar, except that the model and method should emphasize the future rather than the past.

As in any modeling procedure, the first step is to become familiar with the data, in this case, 1983 daily peak demand for Florida Power Corporation. Demand represents the rate at which electric energy is delivered to, or consumed by, a piece of electrical equipment at a given instant of time. At the system level, demand is usually measured as megawatts, i.e., one million watts. The primary source of demand is the energy consuming equipment of the system's customers, and it is the operation of this equipment that directly causes electrical demands to be placed on the system. Thus, system demand fluctuates constantly as consumers turn on and off various electrical appliances, or as the appliances themselves cycle on and off, such as refrigerators and air conditioners. Although demand can theoretically be measured at every instant of time, it is normally reported on an hourly basis as the average demand within each hour. Peak demand, or peak load, refers to the maximum hourly demand recorded for a given period of time. On a daily basis this would be the highest observed demand value out of twenty-four hourly observations. The use of daily peak demand as the dependent variable provides the maximum level of disaggregation for the peak demand data and allows the model structure to concentrate on short-term, day-to-day, impacts for a given year.

For the purpose of modeling peak demands, it is typically assumed that demand consists of two components, a non-weather sensitive "base" demand
which is not influenced by temperature changes and a weather sensitive demand component which is highly responsive to changes in temperature. The base demand component is a function of the usage and stock of non-weather sensitive appliances, while the weather sensitive demand component is dependent upon the usage and stock of weather sensitive appliances. Factors which affect the stock of appliances are generally considered to have long-term impacts, such as changes in life style, technology, incomes, prices, etc. On a daily basis within a given seasonal period, these long-term factors affecting appliance stocks can be assumed to remain constant so that only the short-term factors, those that influence the usage of a given stock of appliances, need be considered. The principle factor that affects the usage of non-weather sensitive appliances is the type of day, i.e., weekday, weekend day, or holiday. Typically, weekdays have similar peak load characteristics; Saturdays have lower demands due to decreased commercial and industrial activity; while Sundays and holidays exhibit even lower demand levels as commercial/industrial activity declines further.

The single most important factor affecting the usage of a given stock of weather sensitive appliances is weather, particularly temperature. Weather plays such a major role because it directly impacts the usage of electric heating and cooling appliances, both of which have very high electric demands. During the winter months, as temperatures drop below comfortable levels, people begin to turn on their heaters, and as the temperature gets colder, more and more people begin operating their heaters thereby increasing the level of demand placed on the system. Likewise, during the summer months, as temperatures climb above comfortable levels, the use of air conditioning drives demand upward. These seasonal weather impacts on peak demand are readily apparent from a graph of daily peak demand data from November 1982 through October 1983, as shown in Figure 1.

One issue that surrounds the development of a model based on weather conditions is the selection of a proper weather variable; for example, should temperature or degree days be used. In the case of peak demands, which occur during a single hour of the day rather than continuously throughout the day, temperature is the logical choice since it provides a measure of weather conditions at a specific point in time. But given that temperature is the appropriate variable to relate with peak demands, several issues remain. First, since FPC serves thirty-two counties along west-central and northern Florida, it is necessary to capture temperature conditions from multiple weather stations for inclusion in the model. This was accomplished by identifying three primary weather stations within the FPC service area and disaggregating that area into three zones surrounding each weather station. Based upon an analysis of electricity sales to each of the three weather zones, a set of weighting factors were developed that measure the percent of total system electricity sales attributable to each weather zone. By applying these weights to their respective weather station's temperature data, a single weighted temperature variable was developed to capture weather conditions along the entire FPC service area.

A second issue concerns the timing of the temperature data and the possibility of lagged effects. As a starting point, the daily temperature variable was defined as coincident temperature, i.e., the temperature value at the hour of peak demand. The temperature during the hours immediately preceding the peak hour would also be expected to influence peak demand, particularly during the summer season when late afternoon thundershowers may cause a drastic decline in temperature at the peak hour, while exerting only a minimal effect on demand. This would imply the need for a temperature variable that is an average, or weighted average, of several hourly temperature values prior to and including the peak hour. The hypothesis of an average temperature variable, rather than coincident temperature, can be tested once an initial model structure is identified.

The major modeling issue that must be resolved before a relatively simple regression model can be developed concerns the non-linear relationship that exists between peak demand and temperature. As noted earlier, during the cool winter months, peak demand is inversely related to temperature. Lower temperatures cause increased usage of heating equipment, which, in turn, cause higher peak demands. In contrast, the summer season shows a positive relationship between peak demand and temperature; higher temperatures yield higher peak demands due to greater air conditioning usage. Theoretically, there also exists a mild temperature range where peak demand is assumed to consist solely of the non-weather sensitive base demand component. Here a change in temperature will not spur any additional heating or cooling and, consequently, has no impact on peak demand. This non-linearity is clearly seen in Figure 2, which presents a scatter plot of FPC daily peak demands versus temperature.

Several options exist in formulating the most useful model structure to account for this non-linear relationship. For simplicity, it was decided to utilize a standard regression model that is linear with respect to the parameters, as opposed to an intrinsically non-linear approach such as is available in PROC NLIN. A quadratic specification was rejected because it would tend toward a symmetrical shape and would, therefore, not allow independent estimates of the winter and summer peak demand-temperature relationship. To this extent it would be desirable to have individual seasonal models that account for the different weather sensitivities of peak demand. On the other hand, a pair of completely independent models would make it more difficult to handle the transitional autumn and spring seasons and ensure consistency between where the two models meet. The solution was to model the entire year of data with a piecewise-linear specification. This approach has the advantage of allowing the peak demand versus temperature relationship to vary between some pre-specified temperature ranges, as well as providing a mechanism for joining the separate pieces.

The fit of a piecewise-linear model is heavily dependent upon the points, or "knots", at which the linear segments join. For the FPC data, a pair of knots were located at temperature values that identify where the winter and summer weather
sensitive portions of demand join the base demand component. As an example of the piecewise-linear form, Figure 3 shows the results of the following regression model.

\[
D_t = a + b_w(T_w-K_w)_+ + b_s(T_s-K_s)_+ + \epsilon_t
\]

where

\[
D_t = \text{Daily peak demand}
\]

\[
T_w = \text{Coincident temperature, if } T_w < \hat{T}_w
\]

\[
= \hat{T}_w \text{ otherwise}
\]

\[
T_s = \text{Coincident temperature, if } T_s > \hat{T}_s
\]

\[
= \hat{T}_s \text{ otherwise}
\]

\[
K_w, K_s = \text{Temperature knot values of 59° & 78°}
\]

\[
a = \text{Base load estimate}
\]

\[
b_w, b_s = \text{Seasonal weather sensitivities of peak demand}
\]

\[
\epsilon_t = \text{Random error term}
\]

MODELS

The basic structure described above represents a starting point for the development and enhancement of a statistical model for daily peak demand. Now that temperature has been incorporated into the model, the initial regression specification requires a couple of dummy variables to account for the effect that different day-types exert on the usage of both weather-sensitive and non-weather sensitive appliances and, consequently, peak demand. Estimation of this regression model revealed that the two dummy variables, one for Saturdays and another for Sundays and holidays, were both significant and yielded expected downward adjustments to peak demand.

An analysis of the residual series discovered several outliers on the summer side of the model which appeared to be caused by the thundershower phenomena noted earlier. In an attempt to capture the impact of temperature conditions prior to the hour of peak, an average temperature variable consisting of the average of five hourly temperatures ending with the peak hour was used in place of coincident temperature. The resultant model eliminated the outlier problem and reduced the mean square error (MSE) of the model by 14%, while the R-square value increased seven percentage points. The existence of previous days temperature impacts were also tested by including lagged average temperature terms into the model. This test revealed that weather conditions for the prior day significantly affected current peak demands on the winter portion of the model, while summer season peak demands were significantly influenced by temperatures during the previous two days. All of the parameter estimates were of the correct sign, laying credence to the theory of heating or cooling build-up effects on peak demand due to several consecutive days of cold or hot temperature conditions.

The regression based model for daily peak demands, however, does not meet all of the standard regression assumptions that underlie a statistically valid model. In particular, the residual series exhibits a strong first-order autoregressive process as detected by a very low Durbin-Watson statistic. The presence of autocorrelated residuals is typically a problem in time series data analysis and, in general, implies an incorrect specification of the functional form of the relationship between peak demand and temperature, or possibly represents the influence of certain important explanatory variables which have been omitted. In this case, it appears likely that the residual autocorrelation may have been introduced by the piecewise-linear form that only approximates the true non-linear relationship. The main consequence of autocorrelated residuals is that they cause the variances of the regression coefficients to be underestimated, thereby inflating t-tests which could lead to the false conclusion that a parameter is significantly different from zero.

Elimination of the autocorrelation problem can be accomplished by moving closer toward the realm of time-series modeling methods available in the SAS/ETS software package. PROC AUTOREG is a procedure specifically designed for the estimation of regression models with autocorrelated residuals. Through a transformation of the data based on an assumed autoregressive error structure, the AUTOREG procedure essentially adjusts and estimates the model while taking into account the autocorrelation. Applying PROC AUTOREG to the daily peak demand model and assuming a first order autoregressive process yielded a 37.5% reduction in the mean square error (MSE), as compared to the regression model of PROC REG. However, the R-square value also experienced a significant decline from 0.861 to 0.815. This seems a paradoxical result! How can model error be decreased without showing a corresponding increase in goodness of fit as measured by the R-square statistic? This is because the R-square is now based on the transformed equation, i.e., adjusted for autocorrelation; therefore, the proportion of variation in peak demands that is explained by the autocorrelation is not included in the calculation of R-square. In general, the value of R-square will decline as the amount of autocorrelation increases, thereby becoming a meaningless value for determining goodness of fit changes from a PROC REG model.

The assumption of a first order residual autocorrelation structure, though popular, is not always a valid assumption. The existence of higher order autoregressive terms are not uncommon, particularly with respect to seasonal time series data. Testing for and incorporating possible higher order terms is easily performed using the BACKSTEP option in PROC AUTOREG, along with a specification of NLAG that is at least equal to the length of any seasonal period. In the case of daily peak demands, with a suspected weekly periodicity, the AUTOREG procedure identified three significant autoregressive parameters at lags one, two and seven. Thus, AUTOREG identified and estimated an autoregressive model for the error structure of the original peak demand model and thereby reduced overall model error.

The fact that AUTOREG models the error process is an important point since that is how it attempts to meet the classical set of assumptions on the error
term which lays the foundation for the statistical adequacy of the regression method. The basic assumption states that the error term is an independent random variable with zero mean and constant variance. In time-series jargon, this defines the error term to be a "white noise" process that consists solely of pure random error. The model has accounted for all significant non-random variations in the data, and there remains nothing left to be modeled. To the extent that the error term can be reduced to white noise, the model is deemed to be an adequate representation of the time series process and may be used for forecasting purposes. It is this aspect of time-series modeling that represents the basis for time-series methodologies such as PROC AUTOREG, Box-Jenkins ARIMA models and the statespace approach.

The Box-Jenkins transfer function method, as contained in PROC ARIMA, is a mixture of causal and time-series techniques and serves as an alternative to AUTOREG. Basically, the transfer function approach models an output variable as a function of one or more input variables plus an error or noise term that represents any variation not explained by the inputs. This is performed using a general class of models known as autoregressive integrated moving-average (ARIMA) models which are capable of adequately describing most time series data in a parsimonious manner. By directly modeling the error term as an ARIMA process, the Box-Jenkins method should provide a model that is at least as good as AUTOREG in terms of goodness of fit, as well as reduce the error term to a white noise process with a minimum number of parameters.

The peak demand model can easily be translated into the Box-Jenkins framework by specifying the same two piecewise-linear weather terms used in the regression equation as the input variables for which transfer functions will be developed. The two day-type dummy variables are also included in the specification and are called intervention variables in the time-series jargon. These intervention variables can be assumed to take on a variety of forms relating to their impact on the output variable. In this instance, a Saturday intervention is assumed to emit a single pulse response which yields an immediate effect on a Saturday peak demand observation. This effect is also assumed to end immediately, rather than slowly tapering off over the next few days. The influence of any lagged temperature variables need not be included as separate inputs to the model since they are inherently included in the ARIMA model structure. Since the Box-Jenkins approach specializes in identifying historical patterns and trends based on lagged values of the output, inputs and noise term, the process evaluates the existence of lagged effects directly from the data. In fact, the general univariate ARIMA specification defines the output variable to be a function of previous values of that output variable, as well as current and past values of a random shock or error term.

PROC ARIMA provides all of the information necessary to perform the three stages of Box-Jenkins modeling, identification, estimation and diagnostic checking. The Box-Jenkins method can only be applied to stationary data, i.e., a time series which exhibits no upward or downward trend component, no seasonal pattern, and no change in variance over time. To overcome this requirement, the IDENTIFY statement provides the tools for identifying and adjusting non-stationary data. It is also the stage where an initial model specification is identified with respect to the type of model, autoregressive, moving-average or mixed, and with respect to the number of lag parameters to include. The ESTIMATE statement yields estimates of the transfer function and noise model parameters for the initial specification. In addition, ESTIMATE facilitates the diagnostic checking stage by providing statistics and tests for analyzing the residuals of the fitted model.

The application of PROC ARIMA to the peak demand data yielded a model structure quite similar to that of AUTOREG. All input variables were significant and of the correct sign. In addition, a one period lag was found to be significant for each of the seasonal temperature inputs. After accounting for the effects of the input and intervention variables, the remaining noise was modeled as a mixed autoregressive moving-average process of order one, plus a seventh order moving average parameter that reflects some weekly seasonality. Based on an evaluation of the standard error estimate of the fitted model, which corresponds to the root mean square error statistic of the regression procedures, the PROC ARIMA results slightly outperformed the model fitted in PROC AUTOREG. However, the results are so close that the average error for the two procedures can, for all practical purposes, be assumed to be identical. For comparison, Table I presents a summary of the root mean square error and R-square statistics for each of the models estimated in this study. That the AUTOREG and ARIMA procedures would yield similar results with respect to average model fit was not unexpected since both explicitly identify and estimate a model for the highly autocorrelated error term. While AUTOREG performed this function with a long autoregression, the Box-Jenkins approach utilized a more general ARIMA process.

Another area of comparison surrounds the parameter estimates for the weather and day-type variables. This information is presented in Table 2, and it also shows that the AUTOREG and ARIMA procedures produce extremely similar estimates of the model coefficients. Based on these results, the choice between modeling peak demands by a regression with autocorrelated errors or by the Box-Jenkins transfer function approach depends less upon statistical validation and goodness of fit criteria than on the specific application of the model, as well as the user's comfort and knowledge of the approach. To the extent that the AUTOREG procedure is simply a variation of the basic regression framework, it may be considered the desired option since it is easier to implement and more explainable than Box-Jenkins methods.

The discussion up to this point has stressed the importance of a white noise error term in time series regression analysis or, equivalently, the reduction of a time series process to a purely random component. However, little attention has been paid to the methods involved in the diagnostic checking and testing of the residual time series in order to determine if they are indeed white noise. The AUTOREG and ARIMA models of daily peak demand, although an improvement over the regression, may not be considered statistically adequate if their
residual series do not represent a white noise process. Two SAS/ETS procedures, PROC ARIMA and PROC SPECTRA provide the methods for analyzing the adequacy of model fit based on an examination of the statistical properties of the residuals.

PROC ARIMA contains the ability to plot the sample autocorrelation and partial autocorrelation functions, along with reference lines at intervals of 2 standard deviations. Plots of these two functions are known as correlograms and represent the tools used in the Box-Jenkins approach for identifying autoregressive moving-average models. A correlogram that reveals no significant autocorrelations, i.e., none beyond two standard deviations, indicates that the series does not contain any dependence or nonrandomness that could be further modeled. Also, a chi-square test statistic that evaluates the correlogram over a series of lags is printed immediately following the correlograms. For an adequate model, this statistic should not exceed the value from a chi-square table at the 10% level. Both of the peak demand models passed these tests of model adequacy in that the residuals revealed no significant autocorrelations.

PROC SPECTRA also contains two tests for white noise, Fisher's Kappa test and Bartlett's Kolmogorov-Smirnov test. These tests are based in the frequency domain of time series analysis and attempt to uncover any periodic components in the residual series that have not been adequately accounted for. The general tool used to search for any hidden periodicities or cycles in the data is the periodogram, which for white noise has a constant expected value. Fisher's Kappa statistic is based on an analysis of the largest periodogram ordinate as compared to the average of all other ordinates to see if that ordinate can be considered as part of a random process. To avoid rejecting the null hypothesis of white noise, the Kappa statistic should be lower than the corresponding table value (see Fuller, p. 288). The residuals of the AUTOREG and ARIMA peak demand models yielded Kappa statistics of 6.498 and 7.157 respectively. Since both of these are under the 5% critical point, approximately 8.027 for this statistic, the null hypothesis that the residuals are white noise is not rejected.

Bartlett's Kolmogorov-Smirnov statistic is based on the normalized cumulative periodogram and, therefore, measures cumulative deviations from the theoretical white noise process. A Kolmogorov-Smirnov test for the Box-Jenkins peak demand model residuals is illustrated in Figure 4, where 5% limit lines are placed around the theoretical white noise line. This test indicates that given a white noise series, the normalized cumulative periodogram would deviate sufficiently to cross the limit lines with only a 5% probability. In PROC SPECTRA Bartlett's Kolmogorov-Smirnov statistic measures the maximum absolute deviation of the cumulative periodogram from the theoretical line, since this would be the point that crosses the limits. Thus, if this statistic exceeds the 5% critical value, the null hypothesis of white noise is rejected. For the residuals of the ARIMA model, Figure 5 shows that the normalized cumulative periodogram never crosses the limit lines and, therefore, the null hypothesis that this represents a white noise process is not rejected. The AUTOREG model was also found to pass this test as Bartlett's Kolmogorov-Smirnov statistic of 0.0534 is well below the 5% critical value of 0.1011.

CONCLUSION

Several alternative methodologies are available within the SAS/ETS library for the purpose of modeling and forecasting time series variables. A graphical analysis of the data and basic regression analysis provided the means of identifying and formulating a piecewise-linear model structure between peak demand and temperature. However, the regression model exhibited strong signs of residual autocorrelation, a typical problem when working with time series data. By directly modeling the residual series, the AUTOREG and ARIMA procedures were shown to provide a drastic improvement in model fit.

In any modeling effort, the statistical adequacy or validity of the model should always be tested based on an examination of the residuals. This study identified and applied two basic tests for model adequacy in PROC ARIMA and two white noise tests in PROC SPECTRA. Since both peak demand models passed all of these tests, they can be deemed adequate and may be used for weather normalization or short-term forecasting purposes. This application found that there was essentially no difference between the AUTOREG and ARIMA results. Consequently, the AUTOREG procedure, which is basically a regression with an adjustment for autocorrelated errors, may be considered the desired option for many analysts since it is easier to use, more explainable to managers, and more popular amongst non-statisticians than the Box-Jenkins ARIMA approach.

*SAS/ETS is a trademark of SAS Institute Inc., Cary, NC, USA.

REFERENCES


### Table 1

**Comparison of Fit Statistics**

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<th>PROC REG</th>
<th>AUTOREG NLAG = 1</th>
<th>AUTOREG NLAG = 7</th>
<th>PROC ARIMA</th>
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<tr>
<td>Root Mean Square Error</td>
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<td>140.2</td>
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<td>R-Square</td>
<td>.861</td>
<td>.815</td>
<td>.834</td>
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### Table 2

**Comparison of Parameter Estimates**

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<th>Weather Parameter Estimates</th>
<th>Day-Type Parameter Estimates</th>
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<tr>
<td></td>
<td>Winter</td>
<td>Summer</td>
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<tr>
<td>PROC AUTOREG (NLAG = 7)</td>
<td>66.0</td>
<td>72.6</td>
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<tr>
<td>PROC ARIMA</td>
<td>64.6</td>
<td>70.7</td>
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### Figure 1

**Daily Peak Megawatt Demands**

*Source: Florida Power Corporation*

**Daily Peak Demand vs. Temperature**

*Source: Florida Power Corporation*

**Normalized Cumulative Periodogram**

*Source: Florida Power Corporation*

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