OPTIMAL ALLOCATION FOR THE COMPARISON OF TWO PROPORTIONS

David C. Huang, The Upjohn Company

I. INTRODUCTION

The determination of sample size needed in a study for testing the difference between two proportions requires the following three quantities.

a. the chance of concluding that the two proportions differ when they are in fact the same (α, Type I error).

b. the chance of declaring that no difference exists when the true difference is non-zero (β, Type II error).

c. the smallest difference which the experimenter would like to declare significant.

Two meaningful comparisons of the two proportions in the study are (i). \( D = \pi_2 - \pi_1 \), and (ii). \( H = p_1' / p_2' \).

For example, the comparison \( D \) can be viewed as the difference between two success rates and the comparison \( R \) has an intuitive appeal since it seems reasonable to say "the success rate is doubled" or "the risk of side effect is halved". Optimal allocation strategies for sample size based on different criteria can result in different recommendations. Armitage (1971) proposed a study design based on a criterion that the estimate of \( D \) or \( R \) has a small standard error. Brittain and Schlesselman (1982) introduced a study design criterion—minimize the variance of \( D \) or \( R \) with respect to \( F \), the fraction of individuals who are sampled from Group 1.

In this paper, a study design for another comparison \( R_d \), the relative difference, where \( R_d = (\pi_2 - \pi_1) / (1 - \pi_1) \), will be examined. The optimization criterion of Brittain and Schlesselman will be applied to minimize the variance of \( R_d \).

II. RESULTS

A. Optimal Precision

Suppose that the frequency of the occurrence of some event (cured, improved) is observed in a random sample of each of the two populations. Let \( r_i \) be the number of occurrences observed among the \( n_i \) individuals in the \( i \)th sample (\( i = 1, 2 \)), and the two proportions are estimated by \( \hat{\pi}_i = r_i / n_i \). Again \( F \) denotes the fraction of individuals who are sampled from Group 1, i.e., \( F = n_1 / N \), where \( N = n_1 + n_2 \).

1. Optimal Precision for \( D \) and \( R \)

Brittain and Schlesselman have shown that:

i. the variance of \( D \)

\[
V(D) = p_1 q_1 / n_1 + p_2 q_2 / n_2
\]

is minimized when

\[
F = \frac{p_2}{p_1 q_1 + p_2} \frac{1}{q_2}
\]

ii. the variance of \( R \)

\[
V(R) = p_1 q_1 / (n_1 p_1 + q_1 / (n_2 p_2))
\]

is minimized when

\[
F = \frac{q_1}{p_1 q_1} \frac{q_2}{p_2 q_2}
\]

Tables 1 and 2 present the percentage increase over minimum in the standard deviations of \( D \) and \( R \) as a function of \( F \). For specified values of \( p_1 \) and \( p_2 \) (i.e., the proportions of individuals who respond in Group 1 or 2), there exist many sampling schemes which are far from the designated optimal value of \( F \) but yield nearly optimal precision of \( D \) and/or \( R \).

2. Optimal Precision for \( R_d \)

The relative difference, \( R_d \), denotes the proportion of individuals among those failing to respond to the first treatment (i.e., Group 1), who would be responding to the second. It is then assumed that

\[
p_2 = p_1 + R_d (1 - p_1)
\]

and the value of \( R_d \) is clearly

\[
R_d = p_2 - p_1 / (1 - p_1)
\]

Fleiss (1981, page 103) presents a more accurate inference about \( R_d \) by taking \( \ln(1 - R_d) \) as normally distributed with a mean of \( \ln(1 - R_d) \) and a variance of

\[
V = p_2 / n_2 q_2 + p_1 / n_1 q_1
\]

where \( q_i = 1 - p_i \) (\( i = 1, 2 \)).

It may easily be derived that

\[
\frac{dV}{dF} = \frac{a}{N(1 - F^2)} + \frac{b}{N(1 - F_2)}
\]

where \( a = p_2 / q_2, b = p_1 / q_1 \)

and

\[
\frac{a}{N(1 - F^2)} + \frac{b}{N(1 - F_2)}
\]
Setting (2) equal to zero, and solving for $F$ and $V$ is minimized when

$$F = -\frac{b}{a + b}.$$  

Table 3 presents the percentage of increase over the minimum in the standard deviation of $R_d$. Again, this table shows that the curve of estimated standard deviation of $R_d$ is relatively flat over a wide range of allocations.

### B. Power Computations

Tables 1-3 show a fair amount of flexibility in selecting a sampling plan that will generate nearly optimal estimation of $D$, $R$, and $R_d$. The problem for experimenters, then, is to determine the effects of different sampling schemes on the power of one test (i.e., the uncorrected chi-square test) for fixed values of $N$, $F$, $p_1$, and $p_2$. The test is two-sided (i.e., looking for a difference in both directions), and the power is the probability of declaring the difference significant when the two proportions are as specified (power = 1-$\beta$).

Two approaches for the power calculations are examined in this paper.

1. **Normal Approximation**

   The power of the uncorrected chi-square test for fixed values of $N$, $F$, $p_1$, and $p_2$ can be obtained by the usual large-sample approximation to the binomial distribution (Brittain and Schlesselman, Equation 3)

   $$Z = \frac{|p_1 - p_2| - Z_{\alpha/2}}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}},$$

   where $p = (n_1 p_1 + n_2 p_2) / (n_1 + n_2)$, and $Z_{\alpha/2}$ and $Z_{\alpha}$ are unit normal deviates corresponding to $\alpha$ and $\beta$.

   Table 4 presents the associated power values with specified values of $p_1$, $p_2$, $N$, and $F$ with $\alpha = 0.05$ are presented in Table 5.

   Tables 4 and 5 show that in most cases the test of highest discriminating power for a given total sample $N = n_1 + n_2$ will be observed when $n_1 = n_2$, i.e., $F = 0.5$. However, the power curves are essentially flat over a wide range of allocations, particularly when the total sample sizes are large. Thus, as when optimal precision is the main goal of a study, it is possible for a sampling scheme intended to maximize precision to yield a reasonable power.

2. **Arcsin transformation**

   The arcsin transformation uses continuous approximation to the binomial distribution in power calculations given the following standard equation (Mace, 1964, pages 101-104)

   $$Z_{arcsin} = \frac{\sqrt{y_1 - y_2}}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} - 2, \alpha} \pm Z_{\alpha}, \quad (3)$$

   where $y = 2 \sin^{-1}(p/2)$.

   The power values based on arcsin transformations for specified values of $p_1$, $p_2$, $N$, and $F$ with $\alpha = 0.05$ are presented in Table 5.

   In addition, Table 5 shows that the power values based on the arcsin transformation are symmetrical about $F = 0.5$, that is about the equal sample size $n_1 = n_2$. For example, the power values for $F = 0.3$ and $F = 0.7$ are the same. The reason for the symmetry is that the sample sizes $n_1$ and $n_2$ enter the equation (3) through $\frac{1}{n_1} + \frac{1}{n_2}$ and $n_1 = F \cdot N$ and $n_2 = (1-F) \cdot N$.

### III. APPENDIX

SAS programs which generate Tables 1-5 are attached. In the SAS programs, ARRAY and DO statements are used for an efficient way of computing the statistics for different combinations of $p_1$, $p_2$, $F$, and $N$. In addition, the following SAS functions are applied for computing the power values.

1. **ARCSIN**: calculates the arcsin transformation.

2. **PROBIT**: inverse normal function. For example, PROBIT(.025) = -1.96 and PROBIT(.975) = 1.96.

3. **PROBNORM**: standard probability distribution. For example, PROBNORM(-1.96) = .025 and PROBNORM(1.96) = .975.

### IV. REFERENCES


The author may be contacted at:
The Upjohn Company
Unit 7293-32-2
Kalamazoo, MI 49001
### Table 1
PERCENTAGE INCREASE OVER THE MINIMUM IN THE STANDARD DEVIATION (SD) OF P1-P2 AS A FUNCTION OF F
(F IS THE PROPORTION OF TOTAL SAMPLE ALLOCATED TO GROUP 1)

<table>
<thead>
<tr>
<th>F</th>
<th>OPTIMAL</th>
<th>P1</th>
<th>P2</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
<th>.9</th>
<th>SD</th>
<th>F</th>
<th>% INCREASE IN SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>120</td>
<td>57</td>
<td>31</td>
<td>16</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>23</td>
<td>0.17</td>
<td>0.69</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.05</td>
<td>98</td>
<td>38</td>
<td>17</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>14</td>
<td>46</td>
<td>0.18</td>
<td>0.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.05</td>
<td>92</td>
<td>40</td>
<td>16</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>14</td>
<td>46</td>
<td>0.73</td>
<td>0.37</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.05</td>
<td>71</td>
<td>26</td>
<td>11</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>23</td>
<td>62</td>
<td>0.94</td>
<td>0.52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.05</td>
<td>76</td>
<td>31</td>
<td>12</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>20</td>
<td>57</td>
<td>0.93</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.05</td>
<td>57</td>
<td>20</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>12</td>
<td>31</td>
<td>76</td>
<td>0.93</td>
<td>0.46</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>0.05</td>
<td>54</td>
<td>18</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>14</td>
<td>33</td>
<td>89</td>
<td>0.89</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td>0.05</td>
<td>40</td>
<td>11</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>20</td>
<td>42</td>
<td>95</td>
<td>0.75</td>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2
PERCENTAGE INCREASE OVER THE MINIMUM IN THE STANDARD DEVIATION (SD) OF (P2-P1)/(1-F) AS A FUNCTION OF F
(F IS THE PROPORTION OF TOTAL SAMPLE ALLOCATED TO GROUP 1)

<table>
<thead>
<tr>
<th>F</th>
<th>OPTIMAL</th>
<th>P1</th>
<th>P2</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
<th>.9</th>
<th>SD</th>
<th>F</th>
<th>% INCREASE IN SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>21</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>17</td>
<td>32</td>
<td>59</td>
<td>123</td>
<td>71</td>
<td>544</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.05</td>
<td>43</td>
<td>13</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>19</td>
<td>40</td>
<td>92</td>
<td>14</td>
<td>719</td>
<td>0.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>34</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>11</td>
<td>24</td>
<td>48</td>
<td>104</td>
<td>11</td>
<td>830</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.30</td>
<td>50</td>
<td>17</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>14</td>
<td>34</td>
<td>62</td>
<td>2</td>
<td>670</td>
<td>0.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>34</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>11</td>
<td>24</td>
<td>48</td>
<td>104</td>
<td>11</td>
<td>830</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>0.60</td>
<td>37</td>
<td>10</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>10</td>
<td>22</td>
<td>45</td>
<td>100</td>
<td>11</td>
<td>759</td>
<td>0.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td>0.70</td>
<td>28</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>13</td>
<td>28</td>
<td>53</td>
<td>113</td>
<td>11</td>
<td>270</td>
<td>0.34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3
PERCENTAGE INCREASE OVER THE MINIMUM IN THE STANDARD DEVIATION (SD) OF (P2-P1)/(1-F) AS A FUNCTION OF F
(F IS THE PROPORTION OF TOTAL SAMPLE ALLOCATED TO GROUP 1)

<table>
<thead>
<tr>
<th>F</th>
<th>OPTIMAL</th>
<th>P1</th>
<th>P2</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
<th>.9</th>
<th>SD</th>
<th>F</th>
<th>% INCREASE IN SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>122</td>
<td>59</td>
<td>32</td>
<td>17</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>21</td>
<td>0.330</td>
<td>0.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.05</td>
<td>92</td>
<td>40</td>
<td>19</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>13</td>
<td>43</td>
<td>0.563</td>
<td>0.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>104</td>
<td>48</td>
<td>24</td>
<td>11</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>34</td>
<td>0.911</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.30</td>
<td>82</td>
<td>34</td>
<td>14</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>17</td>
<td>45</td>
<td>1.471</td>
<td>0.56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>104</td>
<td>48</td>
<td>24</td>
<td>11</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>34</td>
<td>1.577</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.50</td>
<td>104</td>
<td>48</td>
<td>24</td>
<td>11</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>34</td>
<td>2.732</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>0.60</td>
<td>100</td>
<td>45</td>
<td>22</td>
<td>10</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>37</td>
<td>3.225</td>
<td>0.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td>0.70</td>
<td>113</td>
<td>53</td>
<td>28</td>
<td>13</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>26</td>
<td>4.528</td>
<td>0.66</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4
POWER ASSOCIATED WITH SPECIFIED VALUES OF P1, P2, N AND F
FOR AN ALPHA = .05 TWO-SIDED TEST OF THE DIFFERENCE P1-P2
NORMAL APPROXIMATION

<table>
<thead>
<tr>
<th>P1</th>
<th>P2</th>
<th>N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>100</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>200</td>
<td>0.08</td>
<td>0.20</td>
<td>0.32</td>
<td>0.44</td>
<td>0.56</td>
<td>0.68</td>
<td>0.80</td>
<td>0.92</td>
<td>1.00</td>
</tr>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>500</td>
<td>0.03</td>
<td>0.14</td>
<td>0.26</td>
<td>0.38</td>
<td>0.50</td>
<td>0.62</td>
<td>0.74</td>
<td>0.86</td>
<td>1.00</td>
</tr>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>100</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>200</td>
<td>0.06</td>
<td>0.19</td>
<td>0.32</td>
<td>0.45</td>
<td>0.58</td>
<td>0.71</td>
<td>0.84</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>500</td>
<td>0.02</td>
<td>0.14</td>
<td>0.28</td>
<td>0.42</td>
<td>0.56</td>
<td>0.70</td>
<td>0.84</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>100</td>
<td>0.03</td>
<td>0.16</td>
<td>0.30</td>
<td>0.44</td>
<td>0.58</td>
<td>0.72</td>
<td>0.86</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>200</td>
<td>0.01</td>
<td>0.14</td>
<td>0.28</td>
<td>0.42</td>
<td>0.56</td>
<td>0.70</td>
<td>0.84</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>500</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>100</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>200</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>500</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
</tbody>
</table>

TABLE 5
POWER ASSOCIATED WITH SPECIFIED VALUES OF P1, P2, N AND F
FOR AN ALPHA = .05 TWO-SIDED TEST OF THE DIFFERENCE P1-P2
ARCSIN TRANSFORMATION

<table>
<thead>
<tr>
<th>P1</th>
<th>P2</th>
<th>N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>100</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>200</td>
<td>0.08</td>
<td>0.20</td>
<td>0.32</td>
<td>0.44</td>
<td>0.56</td>
<td>0.68</td>
<td>0.80</td>
<td>0.92</td>
<td>1.00</td>
</tr>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>500</td>
<td>0.03</td>
<td>0.14</td>
<td>0.26</td>
<td>0.38</td>
<td>0.50</td>
<td>0.62</td>
<td>0.74</td>
<td>0.86</td>
<td>1.00</td>
</tr>
<tr>
<td>0.05</td>
<td>0.01</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>100</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>200</td>
<td>0.06</td>
<td>0.19</td>
<td>0.32</td>
<td>0.45</td>
<td>0.58</td>
<td>0.71</td>
<td>0.84</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>500</td>
<td>0.02</td>
<td>0.14</td>
<td>0.28</td>
<td>0.42</td>
<td>0.56</td>
<td>0.70</td>
<td>0.84</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>100</td>
<td>0.03</td>
<td>0.16</td>
<td>0.30</td>
<td>0.44</td>
<td>0.58</td>
<td>0.72</td>
<td>0.86</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>200</td>
<td>0.01</td>
<td>0.14</td>
<td>0.28</td>
<td>0.42</td>
<td>0.56</td>
<td>0.70</td>
<td>0.84</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>500</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.25</td>
<td>0.10</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>100</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>200</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>500</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.75</td>
<td>0.50</td>
<td>100</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.75</td>
<td>0.50</td>
<td>200</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.75</td>
<td>0.50</td>
<td>500</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.75</td>
<td>0.50</td>
<td>1000</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
</tbody>
</table>

726
* COMPUTE POWER VALUES -- NORMAL APPROXIMATION

******
data one;
  input pi p2;
cards;
  .05 .01
  .10 .05
  .25 .10
  .40 .30
  .75 .50
;
data yy; set one;
  q1=1-p1; q2=1-p2; pq1=p1*q1; pq2=p2*q2;
  PD=ABS(P1 - P2);
DATA YY; SET YY;
  NN=100; OUTPUT;
  NN=200; OUTPUT;
  NN=500; OUTPUT;
  NN=1000; OUTPUT;
data yy2; set yy;
  ARRAY F(I; F1-F9); ARRAY N1(I; N11-N19);
  ARRAY N2(I; N21-N29);
  ARRAY PP(I; PP1-PP9);
  ARRAY GG(I; GG1-GG9);
  ARRAY NUM(I; NUM1-NUM9);
  ARRAY DEN(I; DEN1-DEN9);
  ARRAY ZB(I; ZB1-ZB9);
  ARRAY POWER(I; POWER1-POWER9);
  DO I=1 TO 9;
    f=round(f, .1);
    N1=F*NN; N2=NN-N1;
    PP=(N1*PI+N2*P2)/NN; GG=1-PP;
    NUM= PD- 1.645*SQRT( (1/N1+1/N2)*PP*GG); 
    DEN=SQRT( (PG1/N1+PG2/N2) );
    ZB=NUM/DEN; 
    POWER=PROBNORM(ZB); 
  END;

PROC SORT; BY PI P2 NN;
PROC PRINT; VAR PI P2 NN POWER1-POWER9;
DATA FINAL; SET YY2; BY PI P2 NN;
  FILE PRINT NOTITLES;
  IF _N_=1 THEN LINK TOP;
  IF FIRST.PI THEN LINK SKIPP;
  PUT PI 34-37 2 P2 41-44 2 NN 47-50
  POWER1 54-57 2 POWER2 60-63 2 POWER3 66-69 2 POWER4 72-75 2
  POWER5 78-81 2 POWER6 84-87 2 POWER7 90-93 2 POWER8 96-99 2 POWER9 102-105 2;
RETURN;
TOP: PUT _PAGE_;
PUT:/// @41 'POWER ASSOCIATED WITH SPECIFIED VALUES OF PI, P2, N AND F'
@41 'FOR AN ALPHA = .05 TWO-SIDED TEST OF THE DIFFERENCE PI-P2'
@41 'NORMAL APPROXIMATION
@41 'F = N1/N2'
@41 'PI  P2'  @35 'N'
@55 '1 2 3 4 5 6 7 8 9'; SKIPP; PUT 2 1 */
/* RETURN;
* COMPUTE POWER VALUES -- ARCSIN. TRANSFORMATION;
* ****************;
da a one;
  input p1 p2;
cards:
  .05 .01
  .10 .05
  .25 .10
  .40 .30
  .75 .50
  
da yYi set one;
  Y1=2*ARSIN(SQRT(P1));  Y2=2*ARSIN(SQRT(P2));
  DIF=ABS(Y1-Y2);
  SIG=0.05;
  * FOR TWO-TAILED, SIG=S1Q*0.5;
  USIG=PROBIT(1-SIG);
  data yY;
  set yY;
  NN=100; OUTPUT;
  NN=200; OUTPUT;
  NN=500; OUTPUT;
  data yY2; set yY;
  ARRAY F(I) F1-F9; ARRAY N1(U N11-N19); ARRAY N2(J) N21-N29;
  ARRAY SQSRN(I) SQSRN1-SQSRN9; ARRAY UPW(I) UPW1-UPW9;
  ARRAY POWER(I) POWER1-POWER9;
  DO I=1 TO 9;
    f=i/10;
    f=round(f, 0.1);
    N1=F*NN;
    N2=NN-N1;
    SQSRN=SQR(N1+1/N2);
    UPW=DIF/SQSRN-
    POWER=PROBNORM(UPW);
  END;
  PROC SORT; BY P1 P2 NN;
  PROC PRINT; VAR P1 P2 NN POWER1-POWER9;
  DATA FINAL; SET YY2; BY P1 P2 NN;
  FILE PRINT NOTITLE;
  IF _N_=1 THEN LINK TOP;
  IF FIRST P1 THEN LINK SKIPP;
  PUT P1 34--37 P2 41-44 NN 47-50
    POWER1 54-57 POWER2 60-63 2 POWER3 66-69 2 POWER4 72-75 2
    POWER5 78-81 2
    POWER6 84-87 2 POWER7 90-93 2 POWER8 96-99 2 POWER9 102-105 2;
  RETURN;
TOP: PUT _PAGE_;
@41 ' TABLE 5'/
@41 'POWER ASSOCIATED WITH SPECIFIED VALUES OF P1, P2, N AND F'/
@41 ' FOR AN ALPHA = .05 TWO-SIDED TEST OF THE DIFFERENCE P1-P2'/
@41 ' ARCSIN TRANSFORMATION'/
@71 'F = N1/N'/
@35 'P1  P2' @49 'N'
@55 '1 2 3 .4 5 .6 7 8 9'
  SKIPP; PUT @1 728  RETURN;
RETURN: