

Colored Candies and PROC MACONTROL: A Sweet Way to Produce Chi-Square Control Charts Part 2: Statistical Presentation

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Colored candies are easy-to-understand tools that can illustrate Statistical Quality Control principles. Each candy bag should have about the same piece counts of each color as the other bags. A multinomial model is useful in characterizing the color piece counts of bags.

Proportion charts (p-charts) and count charts (c/u charts) help assess the stability of the process that fills bags for each color. Maintaining separate p- or c/u-charts to monitor each color, however, is cumbersome and costly. A chart that monitors the colors jointly would be more cost-effective and desirable.

The Chi-square (χ^2) control chart serves to monitor several attributes simultaneously. The χ^2 can test: (a) goodness-of-fit of probability distributions; (b) homogeneity of proportions; and (c) independence of two or more cross-classified variables in contingency tables.

This paper will review the use of the χ^2 control chart to check the stability of piece counts for each color from bag to bag. The chart has other applications as well, such as monitoring defects in manufacturing circuit boards.

Part 1 of this paper examined the use of Base SAS to produce χ^2 control charts. It was found that the FREQ procedure was unable to output cell χ^2 , so the DATA step was used instead. Part 2 deals with the more statistical aspects of χ^2 . This session includes the demonstration of PROC MACONTROL to produce a χ^2 control chart and the DATA step to provide parameters for PROC MACONTROL.

PURPOSE AND PROBLEM

This part continues the paper presented earlier in the Tutorials section. The purpose is to show the use of the Chi-square (χ^2) control chart as a way of monitoring processes like the bag-filling operations of colored candies. In Part 1 we described the bag-filling process and explained how control charts can be used to check the stability of that process. A typical bag of colored candies should have about the same proportional count of pieces of six different colors. To find out what the color distributions for colored candies should be, contact the candy companies (e.g., Mars Inc., Hershey Foods Corporation, etc.).

Suppose the colors in each bag are red, yellow, green, orange, light (tan) and dark brown. Monitoring the process for each color requires six separate control charts, filling walls with costly paper. Others have used single-color pieces to demonstrate statistical process control, e.g., Kohn and Dorner (1992), Kullberg (1992), and Wortham (1992). These examples focused on one color and ignored the other colors. The motivation for considering the χ^2 control chart is to form a composite chart that monitors all the colors while preserving the information of these colors. See Duncan (1950) and Marcucci (1985) for more information. The χ^2 is an old statistical tool used to test (a) goodness-of-fit of probability distributions, (b) homogeneity of proportions, and, (c) independence of (two or more) cross-classified variables in contingency tables. Schlotzhauer (1991) described various χ^2 options within the SAS[®] System that could be used to do goodness-of-fit testing. The same χ^2 used in the above instances can check the stability of piece counts of all colors across bags as they are filled. The χ^2 Control chart has other applications such as monitoring many types of defects during placement of components on complex products like printed circuit boards (PCBs) or printed wiring assemblies (PWAs); mounting MicroElectronics Devices (MEDs) on substrates; and comparing Pareto analyses of the same categories taken over different periods.

PROPOSED METHOD: THE MULTINOMIAL MODEL

Figure 1-1 (in Part 1 of this paper) depicts the colored candies bag-filling operation. Empty bags move along a conveyor and stop at a bag filling station. Colored pieces fill bins for each color. Machines weigh, measure, and fill the bags with colored pieces. Operators or machines pull samples of bags, count the pieces, then enter or plot the data onto control charts. Usually, the sample consists of 20-30 bags according to some continuous sampling plan. Bags showing statistical control on the charts mean the color counts and color mix are stable. These bags may be sealed, packed, and shipped to stores for sale. Charts that lack statistical control mean that some bags may be under or over filled. An investigation should be made to find, fix, or correct the assignable cause(s), e.g., bins empty

of pieces, machines needing recalibration, fixture setups, etc.

Let $j=1,2,3,4,5,6$ index the set of red, yellow, green, orange, tan, and brown color categories of pieces, respectively. The six colors correspond to the $C=6$ response profiles associated with the specific characteristics of interest. Similarly, let $i=1,2,\dots, s=20-30$ index the set of subpopulations (or subgroups) defined with pertinent independent variables. In other words, s is the sample of colored candy bags filled. The subpopulation s could also represent placed components on substrates or the number of periods that the same categories are compared. Finally, let n_{i+} denote total number of colored candies in each bag i ($i=1,2,\dots,s$), where

$n_{i+} = n_{i1} + n_{i2} + n_{i3} + n_{i4} + n_{i5} + n_{i6}$, and n_{ij} = no. of pieces of color j in bag i .

The resulting data can be summarized in Table 2-1:

Table 2-1. (S x C) Response Profile of Color Categories of Counts

BAG	COLOR						SUM
	1	2	3	4	5	6	
1	n_{11}	n_{12}	n_{13}	n_{14}	n_{15}	n_{16}	n_{1+}
2	n_{21}	n_{22}	n_{23}	n_{24}	n_{25}	n_{26}	n_{2+}
↓	↓	↓	↓	↓	↓	↓	↓
i	n_{i1}	n_{i2}	n_{i3}	n_{i4}	n_{i5}	n_{i6}	n_{i+}
↓	↓	↓	↓	↓	↓	↓	↓
s	n_{s1}	n_{s2}	n_{s3}	n_{s4}	n_{s5}	n_{s6}	n_{s+}
SUM	n_{+1}	n_{+2}	n_{+3}	n_{+4}	n_{+5}	n_{+6}	n_{++}

The counts (n_{ij} 's) of Table 2-1 can be also represented as proportions P_{ij} , where P_{ij} denotes the expected proportion for bag i having color j and is represented by Table 2-2:

Table 2-2. (S x C) Response Profile of Color Categories of Proportions

BAG	COLOR						SUM
	1	2	3	4	5	6	
1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{1+}
2	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{2+}
↓	↓	↓	↓	↓	↓	↓	↓
i	P_{i1}	P_{i2}	P_{i3}	P_{i4}	P_{i5}	P_{i6}	P_{i+}
↓	↓	↓	↓	↓	↓	↓	↓
s	P_{s1}	P_{s2}	P_{s3}	P_{s4}	P_{s5}	P_{s6}	P_{s+}
SUM	P_1	P_2	P_3	P_4	P_5	P_6	1.0

The null hypothesis of the χ^2 test of homogeneity is:

$$H_0 : P_{1j} = P_{2j} = \dots = P_{sj} = P_j \text{ where}$$

$$P_j = \frac{\sum_{i=1}^s n_{ij}}{\sum_{i=1}^s \sum_{j=1}^c n_{ij}} = n_{+j} / n_{++}$$

In other words, the χ_i^2 checks if each bag i has the same color proportion p_j of pieces. The Pearson's χ_i^2 is computed as:

$$\chi_i^2 = \sum_{j=1}^c [(n_{ij} - n_{i+} * p_j)^2 / (n_{i+} * p_j)]$$

n_{ij} = observed j th-color count in bag where $i = 1,2,\dots, s$; and P_j = proportion of the j th-color. χ_i^2 has $6-1=5$ degrees of freedom and approximates reasonably well so long as $n_{i+} * p_j$ is not very small (i.e., ≥ 5). If contingency tables have empty cells or small expected cell frequencies (i.e., < 5), then computing exact probabilities is an alternative approach to testing homogeneity of proportions. Fisher's exact test is a method of computing exact cell probabilities and more extreme-than-observed cells for given contingency tables. The method takes changing values of cell frequencies to produce distinct exact probabilities for fixed marginal color totals and bag counts. The size of the summed exact probabilities determines the acceptance or rejection of the homogeneous proportions' hypothesis. Fisher's exact test typically is used with 2×2 contingency tables. Ghent (1972) and Mehta and Patel (1983) described methods extending Fisher's exact test beyond just 2×2 contingency tables. Although their approaches involved extensive calculation, tools within the SAS system facilitate these computations. More could be said about computing exact probabilities of contingency tables. These issues are beyond the scope of this presentation, but are discussed in another SUGI 19 paper by Mehta (1994).

The control limits for χ_i^2 can be set to warning limits (2-sigma) instead of the customary 3-sigma limits. Warning limits increase the sensitivity of control charts and signal early detection of suspicious processes if they do not perform properly. The warning control limits (denoted with the lower-left subscript w) for the χ_i^2 are:

$$wUCL_{\chi^2} = \chi^2 (1-\alpha/2, 5df)$$

$$CL_{\chi^2} = \chi^2 (0.5, 5df)$$

$$wLCL_{\chi^2} = \chi^2 (\alpha/2, 5df).$$

where $\alpha = 0.05$ for 2-sigma warning limits.

The out of control criteria are :

Any two of three consecutive

$$\chi_i^2 < \chi^2 (\alpha/2, 5df) \text{ or}$$

$\chi_i^2 > \chi^2 (1-\alpha/2, 5df)$ signals bag i was overfilled or underfilled of one or more colors. Usually more concern is given to points above the upper control limits than to points below the lower control limits. Points below the lower control limit tend to show more uniformity than expected because all colors are equally present in each bag. This means that:

the P_{ij} 's = $1/6$ ($j=1,2,\dots,6$) and $\chi_i^2 = 0$. Looking for the largest or smallest color cell χ_{ij}^2 contribution tells the analyst, process engineer, or operator which j th-color count (proportion) differed from the expected. Nelson (1982) discusses this procedure in more detail.

Part 1 shows color data and Chi-square control limit calculations of a sample of 20 colored candy bags. Besides monitoring all colors simultaneously, preserving the sequential order of production is another advantage of the χ_i^2 control chart (Figure 1-2 in Part 1). It allows the analyst, engineer, or operator to study runs, trends, and other patterns that might arise in the process. For more information, see Montgomery (1991) and Nelson (1987).

OTHER USES

Other extensions of the χ_i^2 control chart include comparing different Pareto analyses that have similar attribute categories. Kenett (1991) gives a good discussion of how the Chi-square compares Pareto analyses. His χ^2 method may be used to monitor categories of different product (whether they are candy bags, PCBs, or PWAs, etc.) that may span across various machines, operators, or inspectors. This tool helps technical trainers focus training efforts so those defect occurrences (or recurrences) in manufactured product can be reduced.

THE PROGRAM

We tried to create chi-square (χ^2) control charts in SAS/QC[®]. We thought this could be done using m&ms bags' data as follows:

1. Create two data sets:

- To calculate χ^2 values for each bag across the color vectors.
- To calculate the χ^2 control limit parameters that will be used to construct the control chart using the MACONTROL procedure.

PROCs MACONTROL and SHEWHART do not have options to reproduce the χ^2 control charts

we made in Part 1 using Base SAS. We came close to making them, but we were unable to produce the same control limits we got in part 1. The first data set in Step 1 can be created using the SUMMARY procedure (code supplied in Part 1).

The output of the SUMMARY procedure was the χ^2 values for the individual bags. These values would be combined with χ^2 control limits to form another data set that would construct a control chart using the PLOT or GPLOT procedures (in Base SAS[®] and SAS/GRAPH[®], respectively) or the MACONTROL procedure (SAS/QC) for the second data set in Step 1.

When using the MACONTROL procedure, two data sets are needed:

- A data set that will be used with the LIMITS= option that contains the control limits parameters. Let's call this data set CHPARMS. It has the following variables:

VAR - the name of the process variable, e.g., BAGCHI2 (bags' χ^2 variable).

SUBGRP - subgroup variable by bag, e.g., BAGID.

TYPE - tells the MACONTROL procedure whether the process mean and/or the standard deviation are estimated or are standard values. In this paper, **_TYPE_ = STDMEAN** or **STANDARD** where **STDMEAN = CINV(0.5,5)**. See **_MEAN_** below for an explanation of **CINV(0.5,5)**.

LIMITN - the sample size of the subgroup. Since we are dealing with single observations, **_LIMITN_ = 1**.

ALPHA - the risk probability (α) of falsely detecting a process level shift (or the risk of finding a shift that really *does not occur*). Usually **_ALPHA_ = 0.05**.

SIGMAS - the multiple of the standard error of the moving average or EWMA. For warning limits, **_SIGMAS_ = 2**.

MEAN - the process level or mean. In the case of the χ^2 control chart, **_MEAN_ = CINV(0.5,5)**. CINV is the SAS System's inverse chi-square function. With these arguments it calculates the chi-square value associated with a probability of .5 (50%) and 5 degrees of freedom.

STDDEV - the process level/mean standard deviation. According to standard control charting conventions, if we let:

UCLE = $wUCL\chi^2 = CL\chi^2 + 2\sigma$ be the 97.5 percentile,

LCLE = $wLCL\chi^2 = CL\chi^2 - 2\sigma$ be the 2.5 percentile,

and

$CL\chi^2$ is the process center. Then

$CL\chi^2 = \text{CINV}(0.5,5)$,

UCLE = $wUCL\chi^2 = \text{CINV}(.975,5)$,

and

LCLE = $wLCL\chi^2 = \text{CINV}(0.025,5)$.

Therefore, **_STDDEV_** may not be necessary since the standard deviation is contained in the upper and lower control limits when the CINV function was used. **MACONTROL** uses **_STDDEV_** to draw the control limits. **MACONTROL** does not allow suppression of **_STDDEV_** and uses one of four options to compute control limits. We suggest one of three possibilities as an improvement to SAS/QC **MACONTROL** and **SHEWHART** procedures (see "CONCLUSIONS").

SPAN - a constant denoting the number of successive observations that form a subgroup for the moving average. Here we will deal with only single observations, so **_SPAN_**=1.

WEIGHT - the weighting constant assigned to the most recent subgroup of an EWMA/GMA. It takes values between 0 and 1. Here **_WEIGHT_**=1, which says that all of the importance is placed on the most recent data observation and no weight is given to past data. When this situation occurs, the EWMA/GMA reduces to a Shewhart chart.

The **DATA** step that creates the **LIMITS=** option data set (**CHPARMS**) could be as follows:

```
data chparms;
  set ctrllims;
  length _var_ _subgrp_ _type_ $ 8;
  _var_ = 'bagchi2';
  _subgrp_ = 'bagid';
  _limitn_ = 1;
```

```
_alpha_ = 0.05;
_sigmas_ = 2;
_span_ = 1;
_weight_ = 1;
_type_ = 'standard';
_stddev_ = (cinv(.975,5)-cinv(.025,5))/2;
run;
```

```
title 'Chi Square Control Chart:
MACONTROL Procedure';
```

```
proc macontrol data=bagchi2
  table=ctrllims limits=chparms
  graphics;
  ewmachart bagchi2*bagid /
  outtable=outtbl readlimits
  skiplables=3;
run;
```

```
data chparms;
set chparms;
_stddev_ = (cinv(.975,5)-cinv(.5,5))/2;
run;
```

```
proc macontrol data=bagchi2
  table=ctrllims limits=chparms
  graphics;
  ewmachart bagchi2*bagid /
  outtable=outtbl readlimits
  skiplables=3;
run;
```

```
data chparms;
set chparms;
_stddev_ = (cinv(.5,5)-cinv(.025,5))/2;
run;
```

```
proc macontrol data=bagchi2
  table=ctrllims limits=chparms
  graphics;
  ewmachart bagchi2*bagid /
  outtable=outtbl readlimits
  skiplables=3;
run;
```

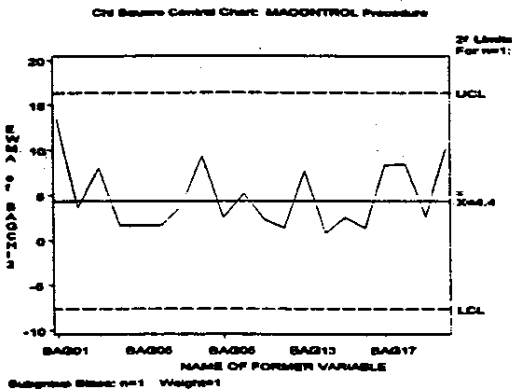
RESULTS

We were unable to obtain the consistent results obtained in Part 1. We found that we could have a correct upper control limit or a correct lower control limit, but not both. The reason for this is that the **MACONTROL** and **SHEWHART** procedures require three parameters: process mean, standard deviation, and subgroup size. This is acceptable for EWMA, UWMA, Individuals, Median, Moving Range, Standard Deviations, and other standard Shewhart charts. For χ^2 , however, it is not acceptable, because standard deviation distorts control limits for the χ^2 control chart. Figures 2-1, 2-2, and 2-3 illustrate our results.

Figure 2-1 was produced when we defined standard deviation like this:

`_stddev_ = (cinv(.975,5)-cinv(.025,5))/2;`

Figure 2-1

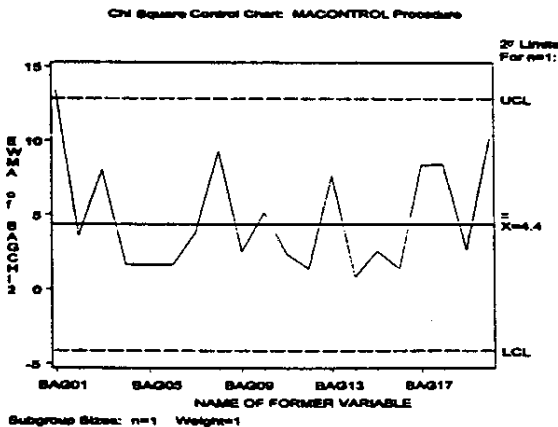


Neither control limit is accurate in Figure 2-1. The upper control limit is too high, while the lower control limit is negative.

We were able to obtain an accurate upper control limit in Figure 2-2, which was produced when we defined standard deviation like this:

`_stddev_ = (cinv(.975,5)-cinv(.5,5))/2;`

Figure 2-2

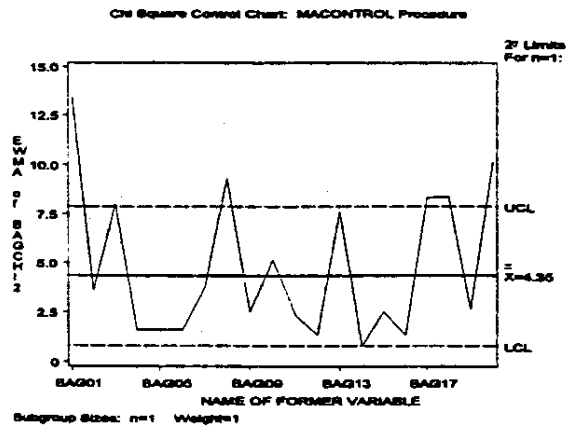


Notice that the upper control limit is correct, while the lower control limit is negative.

To obtain the correct lower control limit shown in Figure 2-3, we defined standard deviation like this:

`_stddev_ = (cinv(.5,5)-cinv(.025,5))/2;`

Figure 2-3



In Figure 2-3, the lower control limit is correct, but the upper control limit is too low.

In a production situation, you could use the results obtained in Figure 2-2, since the upper control limit is used more than the lower control limit. It would be preferable, however, to see both control limits correct.

CONCLUSIONS

We suggest the following as additions to the MACONTROL and SHEWHART procedures:

1. That χ^2 be added to the list of control charts supported by those two procedures.
2. That users be given the ability to suppress specification of the standard deviation in these two procedures.
3. A related suggestion would be to add an option to the SMETHOD= parameter list that tells SAS/QC not to estimate the standard deviation. Currently there are three standard deviation estimation methods in the two procedures, but no option not to estimate it.

The Chi-square control chart presented earlier is versatile, well known (to some extent), and useful for quality control practitioners. By combining the Chi-square with the SAS System, we hope to make this tool applicable, accessible, and palatable (smile)!

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