SETTLING DOMESTICS DISPUTES WITH DESIGNED EXPERIMENTS AND JMP®

Lucia A. Ward-Alexander, Blue Cross Blue Shield of Maryland, Owings Mills, MD
Melvin T. Alexander, Westinghouse, Baltimore, MD

ABSTRACT

One day Mel and Lucia had an argument over the quickest route to drive between their home and a nearby shopping center. Mel suggested that statistics could be used to settle the argument. However, Lucia felt the issue was too complicated to be resolved using statistics because of the numerous factors affecting travel time. Mel countered that argument by pointing out that a statistically designed experiment could be used to settle the issue conclusively.

This paper will show how SAS software (in particular JMP) assisted the couple in answering the key questions posed by the argument. An important lesson learned from this exercise was: statistical thinking not only works well in science, business, medicine, and industry, but it can also improve domestic relationships.

AUDIENCE

The tutorial assumes some knowledge of basic experimental design concepts, e.g., factors; levels; blocking; randomization; replication; and a basic familiarity with JMP software, it's menus and subcommands.

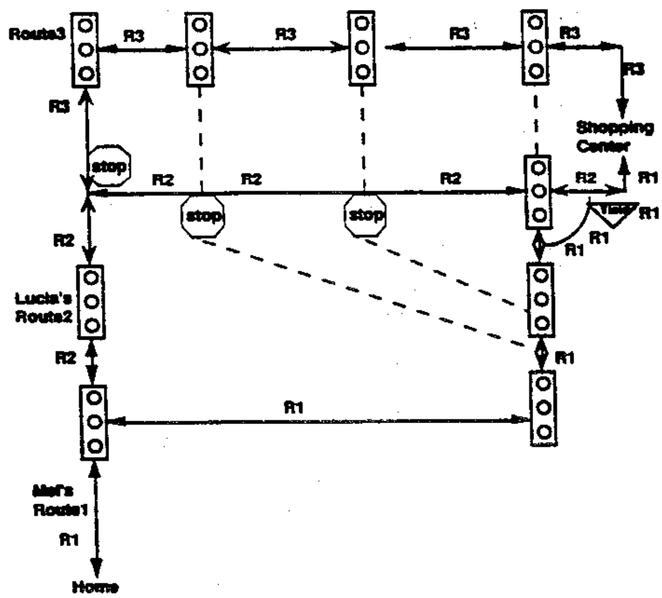
THE SITUATION AND OVERVIEW

The situation began when our son got sick and needed medication from a drug store in a nearby shopping center. There were actually three routes that could be used to get to the shopping center. The argument ensued over which route was quickest. To resolve the situation we asked two questions: Whose route was fastest? If one route was faster, which sets of factors (four main effects and interactions) affects the variability in travel times? JMP software, with its easy user interface and statistical graphical displays, provided the suitable platform for analyzing the data to answer the above questions. Following a discussion of the situation and design considerations is an outline of the steps used in constructing the full factorial design data table in JMP, detailed analysis using JMP, and conclusions.

DESIGN CONSIDERATIONS

We identified four factors. The first factor was the route with three levels. Figure 1 shows the three routes: Route 1 (Mel's route), Route 2 (Lucia's route), and Route 3 ("Control" route). A third route level was added as a control route because it was most commonly traveled by residents and nonresidents of the community where we lived. This "control" route allowed us to compare the travel times of both of our routes with a standard. The second factor was driver set at two levels. Lucia and Mel have different driving styles. The third factor was car. Lucia drove a subcompact while Mel drove a station wagon. The fourth factor was voltime, the amount of traffic volume during different periods of time. During "rush" hours (week days between 7:00 AM - 9:00 AM and 4:00 PM-6:00 PM) traffic volume was heavier than during "nonrush" hour times. Therefore we wanted to take this factor into account.

Figure 1.



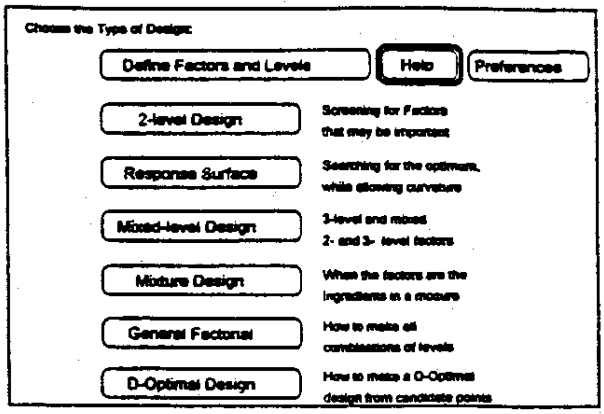
We were restricted in the times we could run the experiments during "rush" hour since we both worked. We decided to let voltime be our blocking factor. The blocking factor accounts for factors of secondary interest that restricts the way in which trips are made for the primary factor of interest (car, driver, and route). Blocking helps to cancel out biases (distortions of experimental results) by carefully selecting the runs of the experiment, i.e., combination of factor levels for the trips. In other words, blocking eliminated taking unnecessary trips across the levels of the factors. Another consideration was to randomize the runs of the experiments. Through randomization, we avoided any systematic, unsuspecting patterns or biases that might occur in taking the trips for the various factor-level combinations.

STEPS IN CONSTRUCTING THE FULL FACTORIAL DESIGN DATA TABLE IN JMP

In order to conduct the analysis, a full factorial design data table was constructed using JMP software. The six steps used in constructing the table are outlined as follows:

Step 1: JMP was launched by selecting the **Design Experiment** command from the Tables menu.
Step 2: The **General Factorial** selection was selected from the **Choose the Type of Design** window (see Figure 2). This allows us to define the factor and up to 10 levels for each factor.

Figure 2.



We also could have chosen the **Mixed-level Design** selection which would have created a design matrix for three 2-level factors and one 3-level factor. However, we did not choose the **Mixed-level Design** because it involves two extra steps: completing the **Define Factors and Levels** window and renaming the middle level of the 3-level factor. When using the **Mixed-level Design,** JMP automatically assigns a value of "0" for the middle level of any 3-level factor. This is the level that would need to be renamed.

Step 3: Identify the factors and define the levels of each factor on the **General Factorial** screen (see Figures 3 -a through 3 - d).

Figure 3 - a.

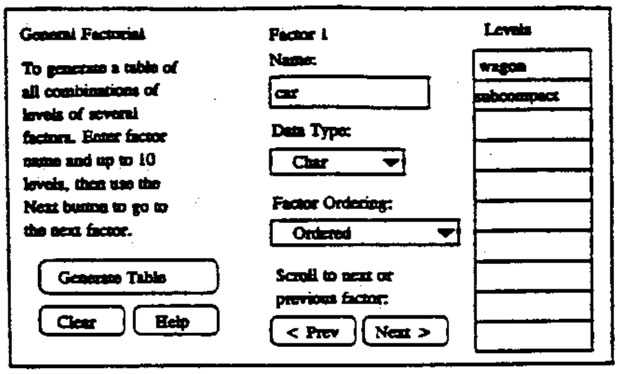


Figure 3 - b.

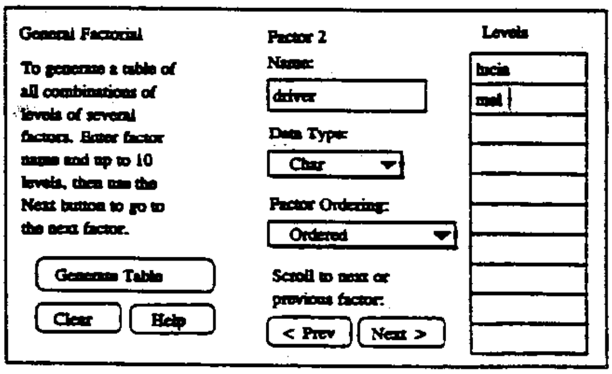


Figure 3 - c.

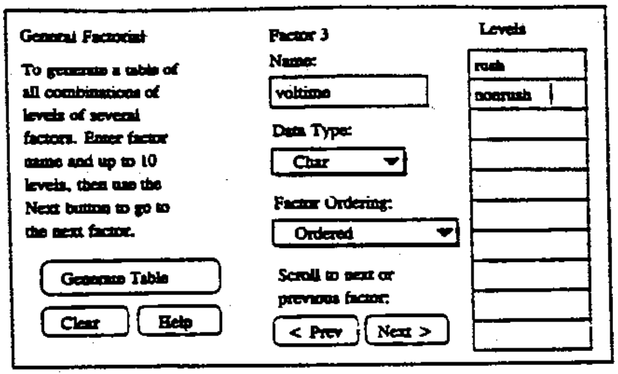
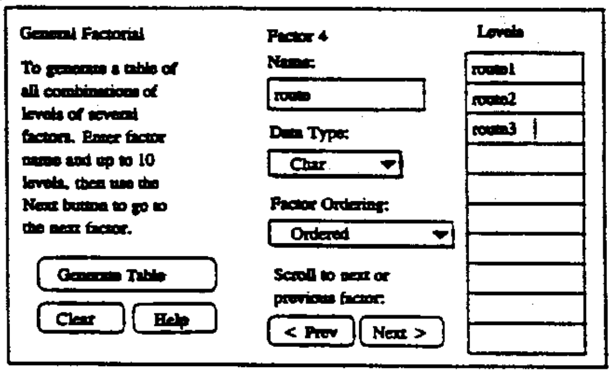


Figure 3 - d.



Step 4: Click on the Generate Table selection from the General Factorial screen, to get a Design matrix. With this experiment, you get a matrix having 24 rows (equal to 2 x 2 x 2 x 3) taking the product of the levels times the factors (see Table 1). Two additional columns are formed, one (Y) which allows for entry of the response-variable values, the other column (Pattern) displays the combination of coded values for the factors in the design. Each digit in the Pattern refers to the level in the corresponding factor. For example, Pattern 1111 refers to the car-wagon, driver-lucia, voltime-rush, and route-route1.

Table 1.

24 rows	Pattern	car	driver	voltime	route	Y
	1121	wagon	lucia	nonrush	route1	•
	1122	wagon	lucia	nonrush	route2	•
	1123	wagon	lucia	nonrush	route3	•
	Etc.					

Step 5: We created another response-variable column to the matrix shown in Table 1 (see Table 2) because we needed two variables for our data analysis: one for the travel time from home to store (h2stime) and one for travel time from store to home (s2htime). The column Y was simply renamed by typing over Y. A second variable column was added by clicking on Cols in the menu bar and naming that column. A third column, mileage, also was added. The data were then entered into the matrix.

Table 2.

Pattern	Car	Driver	Voltime	Rte	h2s time	s2h time	Miles
1111	Wagon	Lucia	Rush	Rte1	8	8	3.1
1112	Wagon	Lucia	Rush	Rte2	8	8	3.2
1113	Wagon	Lucia	Rush	Rte3	9	8	3.3
1121	Wagon	Lucia	Nonrush	Rte1	9	7	3.1
1122	Wagon	Lucia	Nonrush	Rte2	8	6	3.2
1123	Wagon	Lucia	Nonrush	Rte3	11	14	3.3
1211	Wagon	Mel	Rush	Rte1	10	9	3.1
1212	Wagon	Mel	Rush	Rte2	9	8	3.2
1213	Wagon	Mel	Rush	Rte3	8	8	3.3
1221	Wagon	Mel	Nonrush	Rte1	9	8	3.1
1222	Wagon	Mel	Nonrush	Rte2	10	9	3.2
1223	Wagon	Mel	Nonrush	Rte3	9	9	3.3
2111	Subcomt	Lucia	Rush	Rte1	12	8	3.1
2112	Subcomt	Lucia	Rush	Rte2	8	8	3.2
2113	Subcomt	Lucia	Rush	Rte3	9	9	3.3
2121	Subcomt	Lucia	Nonrush	Rte1	8	7	3.1
2122	Subcomt	Lucia	Nonrush	Rte2	7	7	3.2
2123	Subcomt	Lucia	Nonrush	Rte3	8	8	3.3
2211	Subcomt	Mel	Rush	Rte1	7	8	3.1
2212	Subcomt	Mel	Rush	Rte2	10	8	3.2
2213	Subcomt	Mel	Rush	Rte3	9	8	3.3
2221	Subcomt	Mel	Nonrush	Rte1	9	7	3.1
2222	Subcomt	Mel	Nonrush	Rte2	10	8	3.2

Step 6: The Stack Columns command was selected from the Tables menu and h2stime and s2htime were placed into the stacked column frame. We renamed the single stacked column "travel time" as our dependent/response variable. The stacked column gave us replicated

response variables to assess means squared errors properly.

ANALYSES

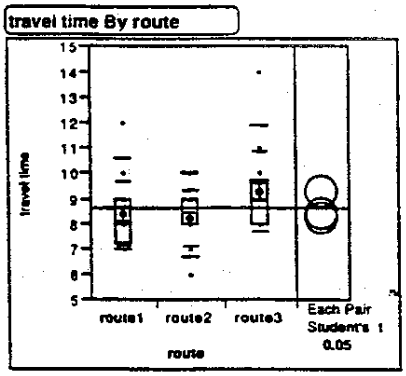
The data analyses consisted of three methods. First, a one-way ANOVA was used to determine if the rates differed significantly using the **Fit Y by X** selection. Next, we used the **Standard Least Squares** selection of the **Fit Model** menu to examine the effect of all the factors on travel time, calculated **Power** for determining probabilities of "false" travel-time differences, and obtained **Least Squares Means** plots for the significant higher-order interactions. Finally, we used the **Fit Model Screening** selection to produce Normal probability and Prediction Profile plots that helped us determine the best factor setting of the fastest travel times.

One-Way ANOVA

We did a one-way ANOVA on the travel time for route using the **Fit Y by X** selection from the **Analyze** menu. From the **Fit Y by X** screen we clicked on route as the X and travel time as the Y. After clicking **Ok** we obtained the travel time **By** route one-way ANOVA table. Figure 4 displays the output of Fit Y by X. The visual one-way ANOVA shows that the mean travel time for routes 1 and 2 do not differ significantly, but route 3 differs significantly from routes 1 and 2. The means and standard deviation's table and tests for equal variance support the visual one-way ANOVA finding. The tests for equal variances show that the variances for the three routes were similar even though the least variation in time was in route 2. This one-way ANOVA does not consider the effect of the other factors.

The Power calculations (see Figure 5) used to detect significant differences in the travel times of the routes for the given sample size, sigma, and p-value (Prob > F = 0.89) were over 78 percent. Although the Power seemed large enough for practical purposes, the amount of explained variation due to the routes was (RSquare) slightly more than 10 percent in the Oneway Anova Summary of Fit (see Figure 6). This suggested that little variation is explained by just route alone. Therefore, we continued to analyze the full factorial model to account for the other factor effect on the travel times.

Figure 4.



Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean
route1	16	8.3750	1.3102	0.3276
route2	16	8.2500	1.1255	0.2814
route3	16	9.2500	1.6125	0.4031

Tests that the Variances are Equal

Level	Count	Std Dev	MeanAbsDif to Mean	MeanAbsDif to Median
route1	16	1.3102	0.9688	0.8750
route2	16	1.1255	0.8438	0.7500
route3	16	1.6125	1.1250	1.0000

Test	F Ratio	DF Num	DF Den	Prob>F
O'Brien[.5]	0.4256	2	45	0.6580
Brown-Forsythe	0.2217	2	45	0.8020
Levene	0.3854	2	45	0.6824
Bartlett	0.9559	2	.	0.3844

Welch Anova testing Means Equal, allowing Std's Not Equal

F Ratio	DF Num	DF Den	Prob>F
2.1481	2	29.414	0.1347

Figure 5.

Power

Alpha	Sigma	Delta	Number	Power
0.0500	1.08066	0.444878	48	0.6838
0.0600	1.08066	0.444878	48	0.7148
0.0700	1.08066	0.444878	48	0.7405
0.0800	1.08066	0.444878	48	0.7624
0.0900	1.08066	0.444878	48	0.7812
0.1000	1.08066	0.444878	48	0.7976
0.1100	1.08066	0.444878	48	0.8121
0.1200	1.08066	0.444878	48	0.8250
0.1300	1.08066	0.444878	48	0.8366
0.1400	1.08066	0.444878	48	0.8470
0.1500	1.08066	0.444878	48	0.8565
0.1600	1.08066	0.444878	48	0.8652
0.1700	1.08066	0.444878	48	0.8731
0.1800	1.08066	0.444878	48	0.8804
0.1900	1.08066	0.444878	48	0.8871
0.2000	1.08066	0.444878	48	0.8933
0.2100	1.08066	0.444878	48	0.8991
0.2200	1.08066	0.444878	48	0.9044
0.2300	1.08066	0.444878	48	0.9094
0.2400	1.08066	0.444878	48	0.9141
0.2500	1.08066	0.444878	48	0.9185

Figure 6.

Oneway Anova

Summary of Fit

RSquare	0.101877
RSquare Adj	0.061960
Root Mean Square Error	1.364226
Mean of Response	8.625
Observations (or Sum Wgts)	48

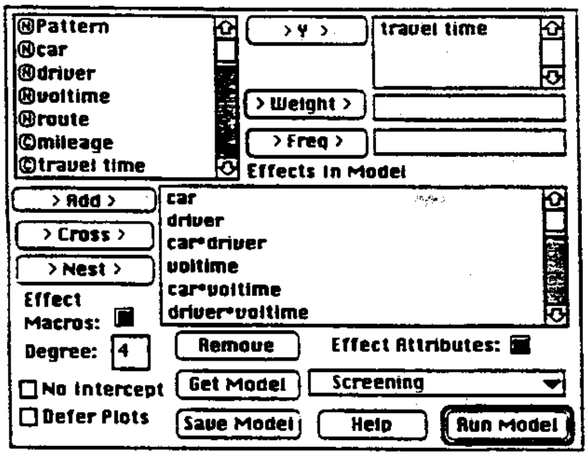
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	9.500000	4.75000	2.5522
Error	45	83.750000	1.86111	Prob > F
C Total	47	93.250000		0.0881

Fit Model

We choose the Full Factorial selection from the Effect Macros option. Using the Fit Model selection from the Analyze menu, we took into account the effect of the other factors. Select travel time as the Y, then select each of the factors to be included in the model. Figure 7 displays the parameters that go into the Fit Model command. Figures 8 through 11 display the output.

Figure 7.



The Summary of Fit table (see Figure 8) showing the full factorial model explained more than 71 percent of the total variation in travel time (RSquare = 0.710456). The significant source variables from the Effect Test table were route, and the voltime*route, driver*route, car*driver* route, car*driver*voltime interactions as noted by the arrows in Figure 8.

Figure 8.

travel time

Summary of Fit

RSquare	0.710456
RSquare Adj	0.432976
Root Mean Square Error	1.06066
Mean of Response	8.625
Observations (or Sum Wgts)	48

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	23	66.250000	2.88043	2.5604
Error	24	27.000000	1.12500	Prob>F
C Total	47	93.250000		0.0130

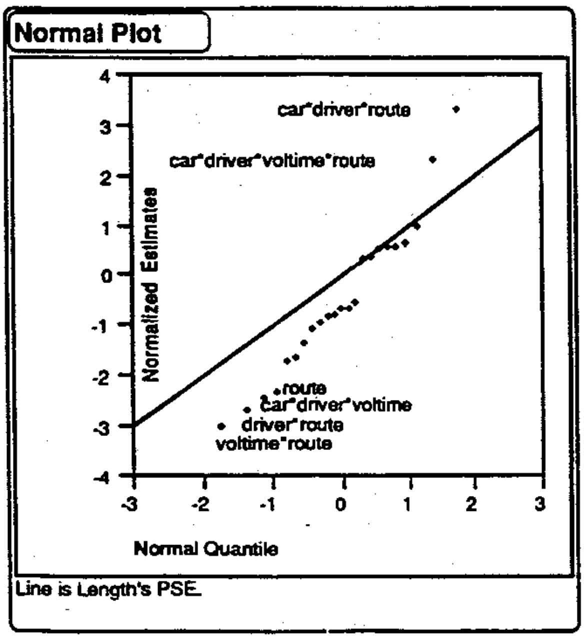
Parameter Estimates

Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F	
car	1	1	0.750000	0.6667	0.4222	
driver	1	1	1.333333	1.1852	0.2871	
car*driver	1	1	0.333333	0.2963	0.5912	
voltime	1	1	0.333333	0.2963	0.5912	
car*voltime	1	1	3.000000	2.6667	0.1155	
driver*voltime	1	1	2.083333	1.8519	0.1862	
car*driver*voltime	1	1	6.750000	6.0000	0.0220	◀
route	2	2	9.500000	4.2222	0.0268	◀
car*route	2	2	0.500000	0.2222	0.8024	
driver*route	2	2	8.666667	3.8519	0.0354	◀
car*driver*route	2	2	12.666667	5.6296	0.0099	◀
voltime*route	2	2	11.166667	4.9630	0.0157	◀
car*voltime*route	2	2	1.500000	0.6667	0.5227	
driver*voltime*route	2	2	1.166667	0.5185	0.6019	
car*driver*voltime*route	2	2	6.500000	2.8889	0.0751	

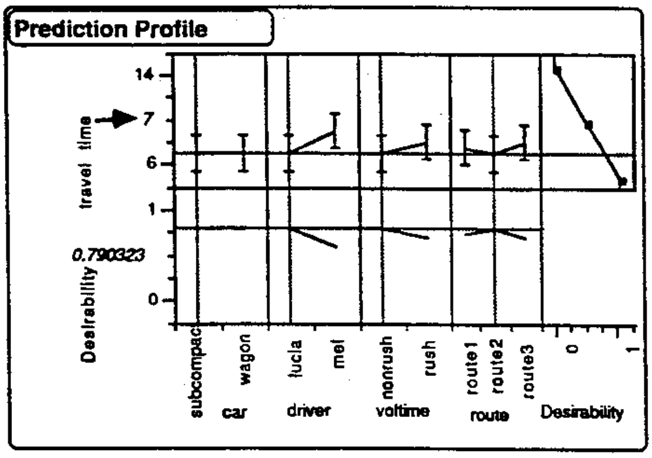
The significant factors identified in the **Effect Test** also stood out with the **Normal Plot** below in Figure 9.

Figure 9.



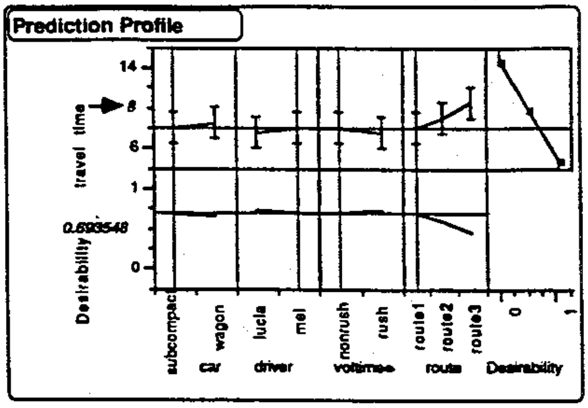
According to the Prediction Profiler (see Figure 10 - a), the best route for Lucia as driver was route 2 with a desirability functional value slightly more than 79 percent and an average travel time of about seven minutes. Desirability functions provide measures of the most desirable or ideal travel times. Desirability functional values range from zero ("worst" or least desirable score) to one ("best" or most desirable score). The Desirability functional value is formed by taking the geometric mean of the Desirability target we set for our response. We wanted to achieve the lowest travel time possible. Therefore, we set our overall Desirability function target to one. Desirability functional values above 63 percent indicate "acceptable and good" travel times according to scales used by Chakraborty and Borah (1989). For more information about Desirability functions, see Harrington (1965), Derringer and Suich (1980), or the *JMP Statistics and Graphics Guide*, pp. 177-183.

Figure 10 - a.



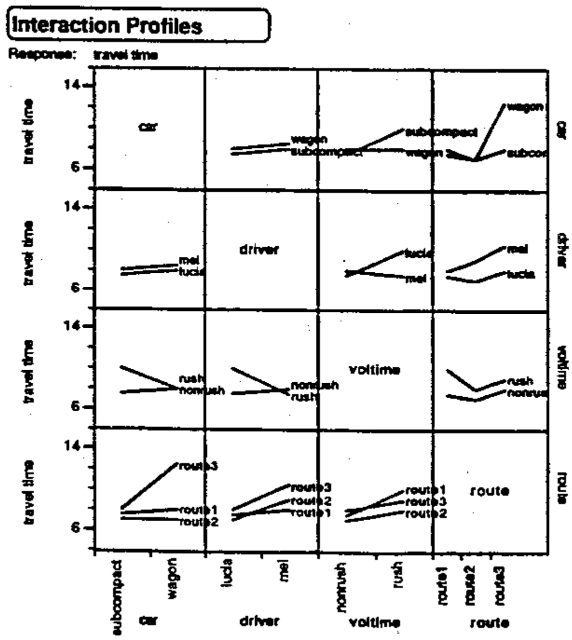
The best route for Mel as driver showed an average of about eight minutes travel time with a Desirability score over 69 percent (see Figure 10 - b).

Figure 10 - b.



The Interaction Profiles plot as shown in Figure 11 allows only two-factor interactions to be compared. The driver*route interaction shows that fewer travel-time variations between drivers were along route 1 than on any other routes. Also, Lucia drove faster than Mel and had fewer travel-time variations across the routes than Mel. Travel time on route 2 (Lucia's route) was lowest in both rush hour and nonrush hours. To compare the significant higher-order interaction effects, we took the **Least Squares Means table from the Standard Least Squares selection of the Fit Model menu.**

Figure 11.



For the car*driver*route interaction, we did the following steps to create a Line Chart as shown in Figure 12:

- We cut the portion of the Least Squares Means table with the scissors tool.

- We copied the screen with the *option*-Copy **as Text** (Alt-Copy as Text under Windows) and *option*-**Paste at End** (Alt-Paste at End under Windows) into a new data table (see **Untitled 1**). We followed the instructions described in the JMP User's Guide, pp. 78-79.
- We added two columns for the lower and upper limits of the Least Squares Means **(LSM)**, LLMSL (=Least Sq Mean - 2 * Std Error) and ULSML (=Least Sq Mean + 2 * Std Error). These columns act like the control limits of statistical control charts. Any minimum travel time below the LLSML, or maximum time above the ULSML, signaled unusually different travel times. Minimum and maximum travel times with a narrow spread of points within the LLSML, a low Least Sq Mean, and ULSML told us which combination of car, driver, and route had the fastest travel time.

Untitled 1

Level	LSM	Std Error	LSML	ULSML
subcompact,lucia,route1	8.75	0.5303	7.6894	9.8106
subcompact,lucia,route2	7.5	0.5303	6.4394	8.5606
subcompact,lucia,route3	8.5	0.5303	7.4394	9.5606
subcompact,mel,route1	7.75	0.5303	6.6894	8.8106
subcompact,mel,route2	9	0.5303	7.9394	10.0606
subcompact,mel,route3	9.5	0.5303	8.4394	10.5606
wagon,lucia,route1	8	0.5303	6.9394	9.0606
wagon,lucia,route2	7.5	0.5303	6.4394	8.5606
wagon,lucia,route3	10.5	0.5303	9.4394	11.5606
wagon,mel,route1	9	0.5303	7.9394	10.0606
wagon,mel,route2	9	0.5303	7.9394	10.0606
wagon,mel,route3	8.5	0.5303	7.4394	9.5606

- We created another data table (see **Untitled 2**) with the **Group/Summary** command of the **Tables** menu using car, driver, and route as **Group** variables. We computed the minimum, and maximum summary statistics for the travel times with the **Stats** dialog.

- We added a new column, Level, using the **Data Source: Formula** feature of the **Calculator Window** to define the car*driver*route interaction. For example, we defined Level with the **Concatenate** argument of the **Character Functions** as follows:

$$car \,\|\, \text{","} \,\|\, driver \,\|\, \text{","} \,\|\, route$$

Untitled 2 By car driver route

Level	Min	Max
subcompact,lucia,route1	7	12
subcompact,lucia,route2	7	8
subcompact,lucia,route3	8	9
subcompact,mel,route1	7	9
subcompact,mel,route2	8	10
subcompact,mel,route3	8	11
wagon,lucia,route1	7	9
wagon,lucia,route2	6	8
wagon,lucia,route3	8	14
wagon,mel,route1	8	10
wagon,mel,route2	8	10
wagon,mel,route3	8	9

The column Level was our matching column that we joined with the **Least Squares Means** table using the **Join** command to create the following file shown in **Untitled 3**:

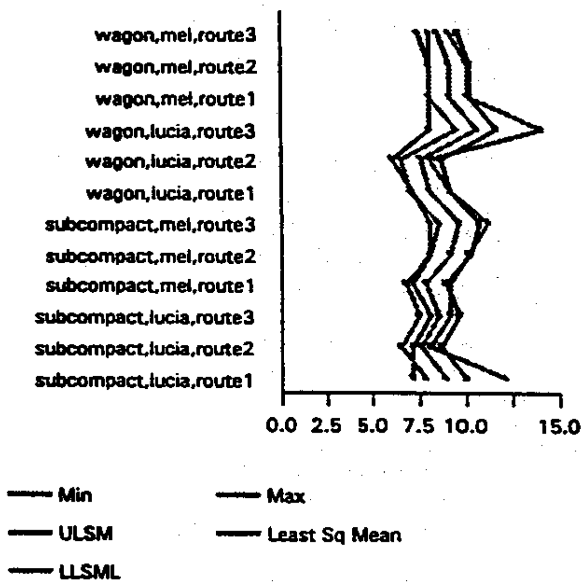
Untitled 3

Level	Min	Max	LSM	Std Error	LLSML	ULSML
subcompact,lucia,route1	7	12	8.75	0.5303	7.6894	9.8106
subcompact,lucia,route2	7	8	7.5	0.5303	6.4394	8.5606
subcompact,lucia,route3	8	9	8.5	0.5303	7.4394	9.5606
subcompact,mel,route1	7	9	7.75	0.5303	6.6894	8.8106
subcompact,mel,route2	8	10	9	0.5303	7.9394	10.0606
subcompact,mel,route3	8	11	9.5	0.5303	8.4394	10.5606
wagon,lucia,route1	7	9	8	0.5303	6.9394	9.0606
wagon,lucia,route2	6	8	7.5	0.5303	6.4394	8.5606
wagon,lucia,route3	8	14	10.5	0.5303	9.4394	11.5606
wagon,mel,route1	8	10	9	0.5303	7.9394	10.0606
wagon,mel,route2	8	10	9	0.5303	7.9394	10.0606
wagon,mel,route3	8	9	8.5	0.5303	7.4394	9.5606

Finally, we used the Overlay Plot command in the Graph menu to produce the Line Chart for the LLSML, minimum, Least Sq mean, ULSML, and maximum travel times below:

Figure 12.

Line Chart of the Car*Driver*Route Interaction



The Line Chart shows that Lucia driving the subcompact along route 2 appeared to have the smallest spread and travel times. The other two car*driver*route interactions with the lowest travel times and spreads were: Mel driving the subcompact along route 1 and Mel driving the station wagon along route 3. Similar plots for the car*driver*voltime and car*driver*voltime* route interactions could be formed as the car*driver*route was produced using the steps described earlier.

SUMMARY AND CONCLUSIONS

We went through several statistical analytical viewpoints to answer the questions: Whose route was faster? Which set of factors affected the variability in travel times? Based on the results Lucia's route was fastest. Travel time variability depended upon certain combinations of car, driver, route, and voltime interactions, e.g., Lucia driving the subcompact on route 2 was faster on the average than her driving the station wagon on route 1. Similarly, Mel driving the station wagon on route 1 was faster on the average than his driving the station wagon on route 2. Relying on just a few analyses would give a partial, incomplete view/picture of the data and would limit our ability to identify of the major components of travel time variability. JMP software provided us with an effective tool to examine the various aspects of the data. JMP led us to a better understanding of what (and how) the factors affected the travel time and its variability.

REFERENCES

Chakraborty, A.K., and Borah, G.K., (1989), "An Application of the Concept of Desirability Functions in Evaluating Erection Quality," *Quality Engineering*, 1, 443-452.

Derringer, G. and Suich, R., (1980), "Simultaneous Optimization of Several Response Variables," *Journal of Quality Technology*, 12, 212-219.

Harrington, E.C., (1965), "The Desirability Function," *Industrial Quality Control*, 21, 494-498.